THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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APRIL 1930

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AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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SPECTRAL ENERGY-CURVE OF SUN-SPOTS[1]

By EDISON PETTIT and SETH B. NICHOLSON

ABSTRACT

Ratio of the energy in the umbra of the spot to that in the photosphere.—This was measured between λ 0.4 μ and λ 2.2 μ by a thermocouple attached to a monochromator mounted in the focal plane of the 150-foot tower-telescope. That portion to the violet of λ 0.4 μ was determined by photographic photometry with an all-speculum optical system, the image-forming mirror being of 60 feet focal length. The mean curve shows that the spot-photosphere ratio varies in a nearly linear manner from 0.21 at λ 0.3 μ to 0.80 at λ 1.7 μ, after which it is nearly constant.

Sources of error.—Depressions at λ 1.38 μ and λ 1.9 μ may possibly be reflected errors due to the deep water-vapor bands at these points. The color-curve of the objective of the tower-telescope being unknown in the red, no corrections could be made for focus. The great focal ratio of this objective (150/1) probably gave sufficient depth of focus to offset this effect partially. The same argument applies to the stigmatic focus of the concave-grating spectrograph used in the measurements to the violet of λ 0.4 μ.

Spectral energy-curve of sun-spots.—This was obtained by multiplying the solar energy-curve for the center of the disk by the spot-photosphere ratios in different wave-lengths. The resulting spectral energy-curve is much like that for the photosphere and corresponds in total energy to a black body at 4860° K.

Temperature of the sun from the solar constant.—The rate of radiation at the center of the disk is 16 per cent greater than that of integrated sunlight, which makes the rate for this point 2.25 cal cm^{-2} min^{-1} and the corresponding temperature from the fourth-power law 5955° K.

Scattered radiation.—The total scattered radiation from the sky and instrument was measured at a point 15″ off the limb of the sun and found to be 0.037 of the intensity at the limb, or, at the center of the disk, 8 per cent of the photospheric light. If the solar energy-curve is assumed to be that of a black body at 5955° K, the energy-curve of the spot fits that of a black body at 4750° K fairly well except to the violet of λ 0.7 μ, where it is in excess. This amounts to 6 per cent of the area of the curve; hence the measured scattered radiation from the sky and instrument is more than double that necessary to explain this excess.

Total energy in the umbra.—This was measured directly with the thermocouple and found to be 0.471 that of the photosphere. The value computed from the curve of the ratio of spot to photosphere is 0.477.

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 397.

Distribution of energy over a spot.—A thermocouple was mounted on a moving plate connected with the registering device. This traced the distribution of energy across the spot. In all cases the energy falls continuously from the photosphere to about half-intensity at the umbra and shows no tendency to increase there. Microphotometer runs across spots with connecting bridges show some of this effect however.

Temperature of sun-spots from water-cell measurements.—The water-cell absorption of a spot is 0.50 mag. for average position on the disk, which corresponds to spectral classification dG7 and temperature 4720° K.

An investigation of the spectral energy-curve of sun-spots between 0.4 μ and 2.2 μ was undertaken at the 150-foot tower-telescope in July, 1922, and completed in September, 1922. The apparatus consisted of a Hilger constant-deviation spectroscope arranged as a monochromator with vacuum thermocouple attached and has already been briefly described.[1] The 17-inch solar image was projected horizontally by means of a plane mirror upon a white cardboard screen over the first slit of the monochromator. A pinhole in this screen admitted light to the slit, making it possible to set an image of a sun-spot or a definite point on the photosphere upon the slit with the electric slow motions of the coelostat. For more accurate guiding during an exposure the whole monochromator was mounted on a double slide so that it could be moved in rectangular co-ordinates in the focal plane, by means of two screws, to follow the spot image while an energy-curve was being traced.

The tangent screw of the monochromator prism table was connected through rods and gearing to a photographic recording device[2] which registered a trace from the D'Arsonval galvanometer connected with the thermocouple. The 7×17-inch plate in the registering device was driven at the rate of approximately 1 inch per minute, an entire trace from λ 0.4 μ to λ 2 μ requiring about 15 minutes.

The general procedure consisted in taking a trace with the slit of the monochromator set on a point of the photosphere just north of the penumbra of the spot, a second trace on the umbra, and a third on a point just south of the penumbra, all curves being recorded on the same plate. In other cases two traces were made on the umbra of the spot and one on the photosphere. Figure 1 illustrates such a record.

The plate was then placed on a comparator and the ordinates were

[1] *Mt. Wilson Contr.*, No. 336; *Astrophysical Journal*, **66**, 43, 1927.

[2] *Journal of the Optical Society of America*, **10**, 267, 1925.

measured in steps of 0.05 μ, and the ratio of the ordinates of the spot-curve to the average of the corresponding ordinates of the two curves from the photosphere or the average of the ordinates of the two spot-curves to that of the curve from the photosphere, as the case might be, was formed for each wave-length. Figure 2 is a plot of these ratios of spot umbra to photosphere for six plates on two spots thus obtained in September, 1922, under good conditions of seeing. The open circles are observations made by Abbot with the bolometer in the Snow telescope in August, 1905.[1] His method consisted in setting the spectrobolometer for a given wave-length and causing the spot to drift across the slit with the lunar gears of the coelostat. The ratios for five wave-lengths were determined from drift-curves traced in this way.

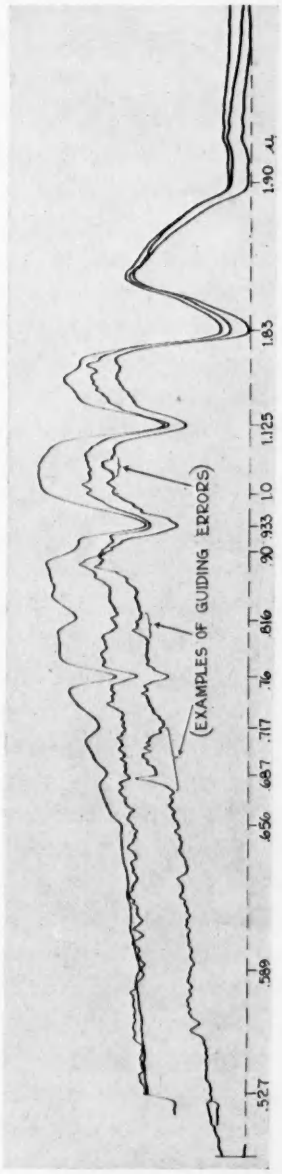

FIG. 1

That part of Figure 2 to the violet of λ 0.4 μ was determined by photographic photometry[2] with a 1-meter concave-grating spectrograph mounted Eagle fashion. A 6-inch concave speculum mirror of 60 feet focal length formed the image for this spectrograph and was fed by one reflection from a speculum-mirror coelostat. The 6-inch solar image fell upon a card in front of the slit, light being admitted to the latter through a pinhole. In some cases a quartz enlarging lens which gave an

[1] *Annals of the Astrophysical Observatory, Smithsonian Institution*, **2**, 233, 1908.

[2] *Mt. Wilson Contr.*, No. 336; *Astrophysical Journal*, **66**, 43, 1927.

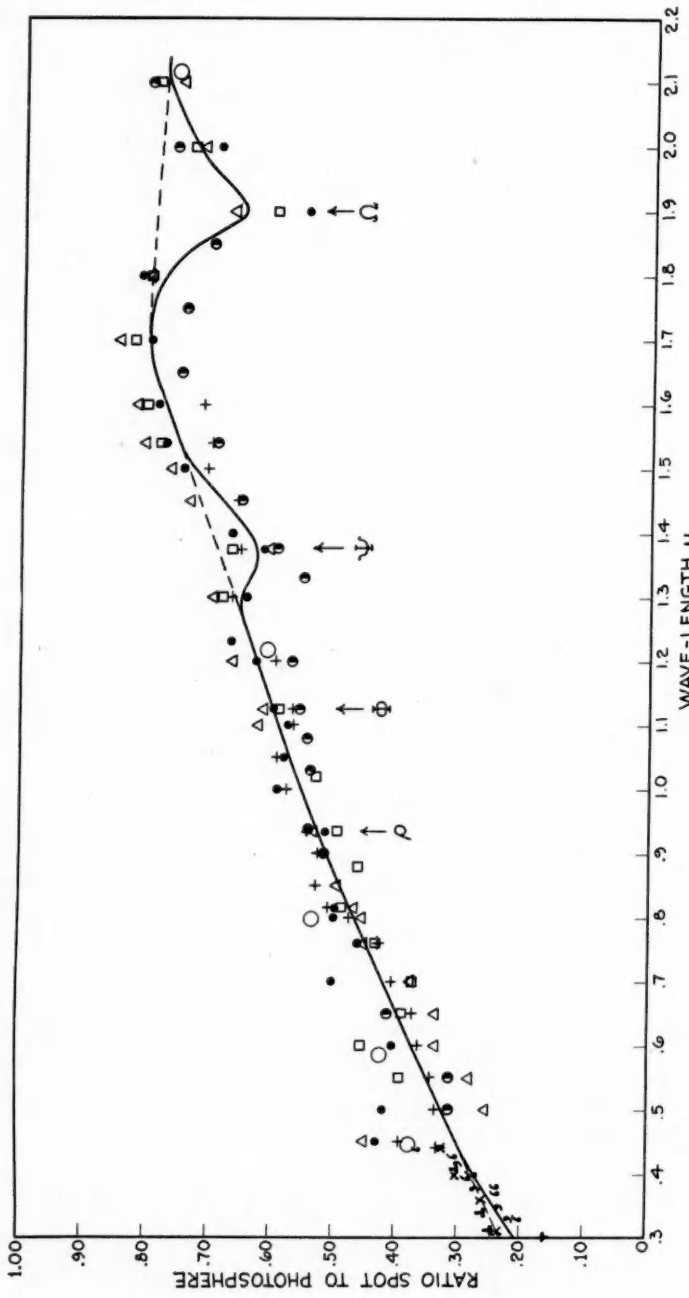
RATIO SPOT TO PHOTOSPHERE
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
0
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
WAVE-LENGTH μ
Ω
Ψ
Φ
ρ

FIG. 2

equivalent focal length of 150 feet was used. The spectrograph was mounted on a stand which permitted slow motion in rectangular co-ordinates in the focal plane for guiding on the spot during the exposure. The method of timed exposures was used with plates and development as already described.[1] When the spectrograph is used in the ultra-violet of the first order the stigmatic focus of the solar image is about 5 cm in front of the slit, but since the ratio of focal length to aperture of the image-forming mirrors is 120 to 300, only a small area about 1 cm in diameter on the grating was used; hence the great depth of focus of the spectrograph made this stigmatic effect practically ineffective. The points shown in Figure 2 to the violet of λ 0.4 μ were obtained on a spot in August, 1927.

The curve in Figure 2 is nearly linear, being slightly convex between λ 0.3 μ and λ 1.7 μ; for greater wave-lengths the ratio, spot to photosphere, appears to be nearly constant. The positions of the atmospheric water-vapor absorption bands are indicated by their symbols. Rather marked drops in the curve will be seen at Ψ and Ω, although none show for the other bands. Whether these are real or merely reflected effects of these deep bands on the measurement of the relatively small quantities is difficult to decide until additional data are available.

Since the color-curve of the objective of the tower-telescope is unknown in the infra-red, it is possible that the curve in Figure 2 to the red of λ 0.8 μ is somewhat too high owing to a mixture of partial pencils of photospheric light in this region with that from the umbra of the spot. No attempt was made to correct the focus, but the great focal-length aperture ratio, 150, of the objective in the tower-telescope probably gave a depth of focus sufficient to offset this effect somewhat. On the other hand, the observations in the infra-red agree well with those of Abbot made with a mirror telescope, and in the visual region run even somewhat less than those of Abbot. The photographic measures in the ultra-violet, which were made with a mirror telescope, show no pronounced discontinuity and check the overlapping radiometric measures between λ 0.4 μ and λ 0.45 μ fairly well. It is hoped that further work may be done with an all-mirror system.

[1] *Ibid.*

Energy-curve of sun-spots.—Curve *A* in Figure 3 is Abbot's solar energy-curve[1] for the center of the disk to which our spot measurements refer. Curve *C* is obtained by multiplying the ordinates of curve *A* by the ratio of the energy in the umbra of the spot to that in the photosphere read from Figure 2 for the corresponding wave-lengths.

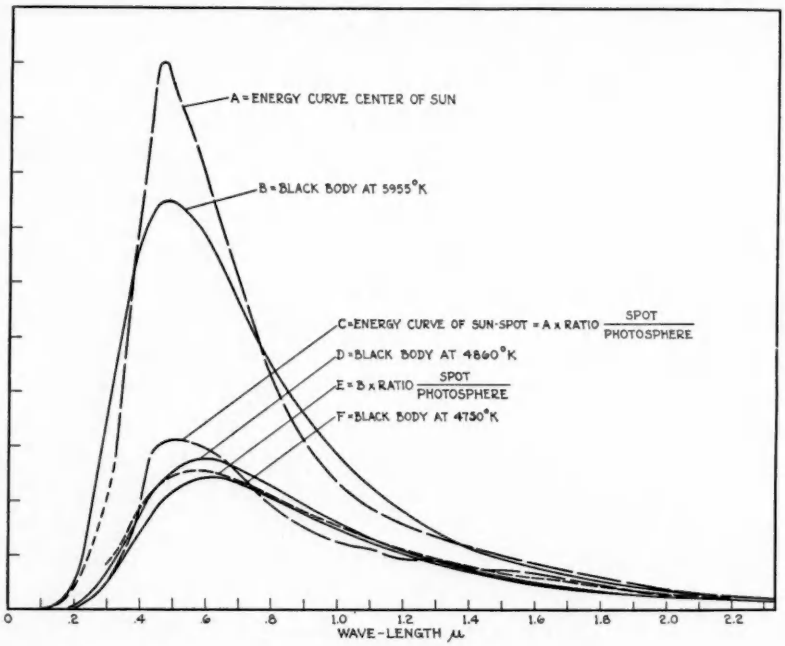

FIG. 3

That the energy-curve *C* of a sun-spot is not that of a black body is seen by the following argument. The ratio of the area of curve *C* to curve *A* should be the ratio of their total energies and is 0.445. The total energy of integrated sunlight is the solar constant, 1.94 cal cm^{-2} min^{-1}. The rate of radiation from the center of the disk where the sun-spot measurements were made is 1.16 times this quantity,[2] or 2.25 cal cm^{-2} min^{-1}, which corresponds to a black-body temperature of 5955° K. This is the temperature of the photosphere given by total radiation measurements and would appear to

[1] *The Sun* (1929), p. 107; *Annals of the Astrophysical Observatory, Smithsonian Institution*, **3**, 159, 1913.

[2] *Mt. Wilson Contr.*, No. 299; *Astrophysical Journal*, **62**, 210, 1925.

be the best value of the temperature of the sun derived from the solar constant. If the spot radiates like a black body, its temperature is therefore

$$T = 5955^\circ \text{ K} \times \sqrt[4]{0.445} = 4860^\circ \text{ K}.$$

Curve *D* in Figure 3 is a black-body energy-curve for this temperature having the same area as the sun-spot energy-curve *C*. It appears from curves *D* and *C* that, generally speaking, the sun-spot has more energy than the black-body curve for wave-lengths to the violet of λ 0.7 μ, and correspondingly less for wave-lengths to the red. A considerable part of the deviation of the sun-spot energy-curve *C* from a black-body curve such as *D* is due to the deviation of the solar energy-curve *A* from the black-body curve. To demonstrate this the black-body curve *B* for 5955° K has been drawn with the same area as the solar energy-curve *A*, and from it the energy-curve *E* has been drawn by multiplying *B* by the ratio, spot to photosphere, as was done to obtain curve *C*. It will be seen that this curve lies for the most part between the black-body curve *F* for 4750° K and *D* for 4860° K, agreeing to some extent with the former in wave-lengths greater than about 0.7 μ.

Scattered photospheric light.—A comparison of the spectra of photospheric light with those of sun-spots used to obtain the ultra-violet portion of the curve in Figure 2 shows little difference between the two in the region below the calcium lines H and K. This suggests that the spot spectrum has superposed upon it a considerable amount of photospheric light which would in turn explain the deviation of the energy-curve *E* of the spot spectrum (computed from the black-body curve *B*) in the visual and ultra-violet region from the energy-curve *F* of a black body at 4750° K. The area between these two curves to the violet of λ 0.7 μ is only 0.06 of the area of curve *E;* hence it might be thought that this represents the order of magnitude of the total scattered photospheric radiation. This scattered photospheric light may be from the sky, the instruments, and from the spot itself, where the photospheric light projected into the spot is scattered by the gases there to the observer along with the intrinsic light of the spot.

The total scattered radiation in the instrument and sky was

measured at the limb of the sun with the thermocouple behind a pinhole diaphragm as described below. The radiation from the sky and the instrument 15″ outside the limb of the sun was found to be 0.037 of that 15″ inside the limb of the sun. In order to approximate the conditions of scattering in a spot we may imagine another sun placed on the other side of the point, 15″ away, so that it would be practically surrounded by the two solar disks, whence the scattered light would have been 0.074 of the intensity 15″ inside the limb. After allowing for the increase in scattered light toward the center of the disk it appears from this rough estimate that scattering in the earth's atmosphere and the telescope will amount to fully 8 per cent or more of the photospheric radiation or 16 per cent of the total radiation received from the spot, which is approximately half that of the photosphere, as will be shown shortly. The radiation scattered by the sky and instrument is therefore more than twice as much as that necessary to explain the deviation of curve *E* from curve *F*.

Total energy in the umbra.—We may measure the whole radiation in the spot directly with the thermocouple and compare the result with that obtained from spectral distribution of energy. For this purpose the thermocouple was placed directly behind a pinhole in a card and mounted in the focal plane of the 150-foot tower-telescope. With this arrangement the ratio of the whole radiation received from a small area of the photosphere on each side of the spot was compared with the same area of the umbra. From twenty measures on six different spots in October, 1921, this ratio was found to be 0.471. The greatest value was 0.521 and the least 0.428. The solar energy-curve at the earth's surface on Mount Wilson was then multiplied by the transmission of the objective and the reflecting power of the silvered mirrors of the tower-telescope, ordinate by ordinate, and the resulting area compared with that obtained by multiplying this same curve by the ordinates of the curve in Figure 2. The ratio of the latter to the former, which should be the ratio of the total energies in the spot and the photosphere, is 0.477, which agrees quite well with that (0.471) determined above from direct measurement. The very small pinhole, approximately 0.01 mm in diameter, which could be used in these measurements made the check the more rigorous, since the aperture used over the slit of the

monochromator was approximately 0.5 mm in diameter, which gave a usable slit about 0.5×0.1 mm.

In order to study the distribution of total energy over the spot, the thermocouple with its pinhole diaphragm as used above was mounted on a screw-driven plate which was connected with the photographic registering device by a rod and gears, as in the case of observations of the spectral energy-curve. In this manner the thermocouple with its pinhole diaphragm was made to move across the spot

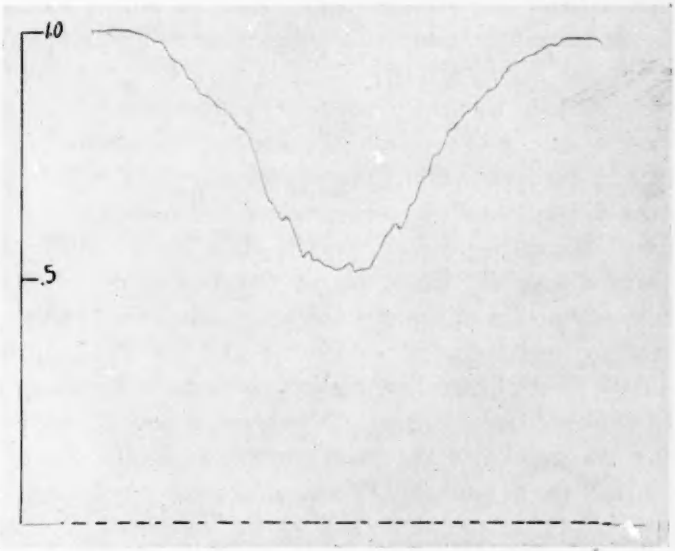

FIG. 4

while the galvanometer traced the corresponding distribution-curve in the registering device. Figure 4 is a reproduction of such a trace.

It will be noted that in Figure 4 the total energy falls continuously from the photosphere throughout the penumbra to the umbra, where it remains fairly constant at about half the photospheric intensity, as was found from the deflections described above. It will be recalled that Secchi[1] believed that the penumbra actually became brighter toward the periphery of the umbra, which his drawings strongly represent, and explained it by the convergence of the penumbral filaments toward the umbra.[2] None of our curves shows

[1] *Le soleil*, 1, 81, 1875.

[2] Russell, Dugan, and Stewart, *Astronomy*, 1, 200, 1926; Young, *The Sun* (1910), p. 116.

this effect, and considerable experience in drawing the daily polarity charts at the 150-foot tower gives visual confirmation of this result. Secchi's observation is possibly an effect of contrast. Some microphotometer runs made on a photograph of spots crossed by penumbral bridges show, however, something of the effect along the bridge. How much photographic contrast is involved is difficult to estimate.

The temperature of a sun-spot from water-cell measurements.—The water-cell absorption of the umbra of a spot near the center of the disk measured at the 150-foot tower-telescope is 0.47 mag., while the photosphere at the center of the disk gives 0.38 mag. The water-cell absorption for integrated sunlight[1] with two reflections from silver, but without the lens of the tower-telescope, is 0.41 mag. The difference of 0.03 mag. is probably due to the difference between the center of the disk and integrated sunlight. This would make the water-cell absorption for spots in an average position on the sun 0.50 mag., the corresponding spectral classification[2] dG7, and the temperature 4720° K. That the water-cell absorption of a spot should be affected in this manner by its position on the solar disk is indicated by measurements made on a spot in solar longitude 75° west of the central meridian and 8° north latitude, which gave a water-cell absorption of 0.7 mag. Such measurements are difficult, however, on account of the steep energy distribution-curve of the photosphere at the limb of the sun, and further observations will be necessary to determine accurately the water-cell absorption of a spot so near the limb.

We have finally the following comparative temperature data:

Temperature of the photosphere at center of disk	5955° K
Temperature of sun-spot umbra from spectral distribution of energy	4860°
Temperature of sun-spot umbra from spectral distribution of energy corrected for scattered light	4750°
Temperature of sun-spot umbra from water-cell absorption	4720°

From these data it appears that the temperature of the umbra of a sun-spot is probably near 4800° K, or possibly a little less.

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MOUNT WILSON OBSERVATORY
January 1930

[1] *Mt. Wilson Contr.*, No. 369; *Astrophysical Journal*, **68**, 289, 1928.

[2] *Ibid.*, p. 297, Table V.

A METHOD FOR DETERMINING THE DISTRIBUTION OF STELLAR ABSOLUTE MAGNITUDES FROM THE DISTRIBUTIONS OF ANGULAR AND LINEAR PECULIAR VELOCITIES[1]

By GUSTAF STRÖMBERG

ABSTRACT

The method described was devised for the purpose of finding the *distribution of stellar absolute magnitudes* with high *accuracy* and with the greatest possible "*resolution*," however complex this distribution may be. It avoids use of analytical distribution functions, as well as means and dispersions, and is *inductive* and entirely *numerical*. The procedure depends on a knowledge of the distributions of reduced τ-components and peculiar radial velocities V' for the group of stars studied. The distributions are converted into logarithmic distributions, account being taken of the effect of errors, which is greatest in the case of the smallest values. The new variables are $x=\log \tau + 0.2m$ and $y=\log V'-1.6756$. Each distribution is defined by a sequence of frequencies within constant intervals. The unknown distribution of absolute magnitudes is determined by an integral equation. To solve this an approximate distribution of absolute magnitudes M is combined with that of y to give a computed distribution of x. The differences between the observed and the computed distributions of x are formed and used to derive by least-squares corrections to the assumed distribution of M. The corrected distribution of M is again combined with the distribution of y, and the computed distribution of x compared with the observed distribution. The process is repeated until a satisfactory reproduction is obtained of the distribution of x.

1. In the derivation of stellar absolute magnitudes we may distinguish between individual and statistical determinations, the former giving the absolute magnitude for a particular star with a certain probable error, which may be small or large, the latter usually giving the mean absolute magnitude for a group of stars.

Individual determinations are possible in the following cases: (1) when the trigonometric parallaxes can be determined with small percentage errors; (2) when several stars in a group have strictly parallel motions and their proper motions and some of their radial velocities are known; (3) when an observable physical characteristic of the star is correlated with its intrinsic brightness, the accidental errors being dependent upon the degree of correlation; (4) when the stars in a group are all at approximately the same distance (on a logarithmic scale). In the third and fourth cases it is necessary to determine certain constants in the reduction formulae either from

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 395.

individual stars or by statistical methods, and any errors in these constants will introduce systematic errors in the absolute magnitudes.

The statistical methods in use are based on (1) the determination of mean trigonometric parallaxes; (2) comparison between angular and linear motion, usually a comparison between their means (parallactic motion) or their dispersions (mean peculiar motion).

In most cases, however, the ultimate purpose is not to determine the absolute magnitude of a particular star or the mean absolute magnitude of a group of stars, but rather the distribution of absolute magnitudes within a group, which may contain stars of a certain spectral type or the whole galactic system, and which may or may not be confined within definite limits of apparent magnitude. Particularly since the discovery of giants and dwarfs, many attempts based on both individual and statistical determinations have been made to find in a unique way the distribution in absolute magnitude among groups of stars. Much success has been attained by Kapteyn and van Rhijn,[1] who have combined and extrapolated the data for a number of smaller groups, usually the mean absolute magnitudes and estimates of the spread, into a distribution covering the whole galactic system. But any introduction of means and of "spread" or "dispersion" may obliterate the finer details of the absolute-magnitude distribution. These details are very important since they may give information on the existence of separate maxima and minima, which may be expected if the actual absolute magnitudes correspond to different states of stable equilibrium of radiation. A method capable of giving the absolute-magnitude distribution with a certain degree of "resolution" is accordingly needed. It might be thought that individual absolute-magnitude determinations would be best suited to such a study, but that is not necessarily the case, since the errors in the determinations, particularly in method (3) above, may tend to smooth out less prominent maxima and minima—a tendency which, of course, is not removed by increasing the number of stars. A particular aim of the present study has been to ascertain if the supergiants are separated from the ordinary

[1] *Publications of the Kapteyn Astronomical Laboratory*, No. 38, and previous publications.

giants by a distinct frequency minimum, or if the distribution shows a certain amount of asymmetry, as is indicated by the spectroscopically determined absolute magnitudes.

2. If we may regard the absolute-magnitude distribution as similar to that represented by a normal error function, the dispersion in absolute magnitude may be determined from the "excess" in the distribution of peculiar angular motions, as has been done by Gyllenberg.[1] It is also possible to determine the constants of two error-curves, which has also been done by Gyllenberg[2], but the determination, which is based on the method of moments, is very uncertain, for reasons which will be explained later.

The present method developed from an attempt to find if the distribution of absolute magnitudes is a definite function of the distribution of angular and linear tangential motions, the angular motion being reduced to a constant apparent magnitude. Since, under certain conditions, the distribution of linear tangential motions may be replaced by that of radial velocities, the main problem is to handle complex distributions properly without introducing simplifying assumptions which may obliterate important features of the unknown absolute-magnitude distribution. Complex distributions have been studied, particularly by Charlier, Wicksell, Gyllenberg, Malmquist, and others at the Lund Observatory by the use of moments, and many valuable results have been obtained. But it is clear that a distribution with many characteristic features can be analytically described only by the use of many parameters. To determine these it is necessary to compute moments of high order, which obviously are extremely uncertain because they involve means of observed quantities raised to a high power. Even with moments of third and fourth order there is always doubt whether the outstanding values ought to be included or not. The effect of the inclusion of a single star may be overwhelming.

3. The method here developed consists in relinquishing all attempts to represent the distributions involved by analytical func-

[1] *Meddelanden från Lunds Observatorium* (2d ser.), No. 41*a*, Charlier Festskrift, 1928. See also Strömberg, *Mt. Wilson Communications*, No. 103; *National Academy Proceedings*, **14**, 834, 1928.

[2] *Meddelanden från Lunds Observatorium* (2d ser.), No. 41*b*, Charlier Festskrift, 1928.

tions, at least until the distributions are known numerically, and in characterizing each distribution simply by a sequence of numbers representing the number of values occurring within definite intervals. The whole computation is thus strictly numerical; the observations are left free to speak for themselves with the least amount of interference from the investigator. The only interference introduced is that of arranging the observed values in intervals of rather narrow class breadths and of assuming a certain degree of continuity in the distributions. All moments are regarded as artificial and thus to be avoided; even the use of means is prohibited except within definite narrow limits when their introduction has no tendency to smooth out unduly the unknown distribution.

The problem is simplest in the case of peculiar motions, and the method has so far been developed only for this case. It has been applied to the determination of absolute magnitudes among stars of spectral classes K and M and of apparent magnitude brighter than 6.0. The results will be published in the following *Contribution*, No. 396.

4. The proper motion, supposed to be known for each star, is projected on a great circle passing through the star and the antapex, and on another great circle perpendicular to the first. The latter projection is the τ-component and is the only part of the proper motion used in the present investigation. It is expressed in seconds of arc per year. The radial velocities of the stars are also supposed to be known, but it is not necessary that radial velocities and τ-components be known for exactly the same stars. The main thing is that the distribution of the τ-components and of the radial velocities be representative for the stars studied.

For any star the relation between parallax π, apparent magnitude m, and absolute magnitude M is

$$M = m + 5 + 5 \log \pi , \tag{1}$$

which defines the absolute magnitude as the value of the apparent magnitude at a distance of 10 parsecs. The relation between the angular tangential motion τ and the linear tangential motion T in the direction of τ is

$$k\tau = T\pi , \tag{2}$$

where k is dependent on the units employed. T is expressed in kilometers per second; numerically, therefore, k is the ratio of the mean distance earth-sun, expressed in kilometers, to the number of seconds in a year, whence

$$k=4.738 \text{ km year/sec.}$$

Eliminating π from equations (1) and (2), we find

$$M=m+5 \log \tau-5(\log T-\log k-1)\,, \tag{3}$$

or, expressed in other terms,

$$M'=x-y\,, \tag{4}$$

where

$$\left.\begin{aligned} M'&=0.2\,M\,,\\ x&=\log \tau+0.2\,m=\log \tau'\,,\\ y&=\log T-\log k-1=\log T-1.6756\,. \end{aligned}\right\} \tag{5}$$

The quantity $\tau'=\tau 10^{0.2m}$ is often referred to as the reduced τ-component.

We have now to deal, not with individual stars, but with three distribution functions, viz., those of x, y, and M', denoted by $F_1(x)dx$, $F_2(y)dy$, and $F_3(M')dM'$, respectively. The velocities T are not known; but, being one-dimensional, that is, projected on axes independent of the direction of motion of the individual stars, they are statistically comparable with the peculiar radial velocities. If the peculiar velocities depend on the direction of motion, as, for instance, in an ellipsoidal velocity distribution, the stars must be well distributed over the sky. If this condition is fulfilled, we may thus replace the distribution of T by that of the peculiar radial velocities V', and thereby determine the distribution $F_2(y)dy$.

5. The distribution $F_1(x)dx$ and $F_2(y)dy$ must first be corrected for the effect of errors in the τ-components and in the peculiar radial velocities V', which is very great when τ' and V' approach zero. Although rather obvious, it may be mentioned here that these quantities are to be used without regard to signs. The probability of infinite values of x and y must be infinitely small, because a value of τ' or V' exactly equal to zero has an infinitely small probability.

For the intervals $d\tau'$ and dV' must then also be zero and the probabilities $F(\tau')d\tau'$ and $F(V')dV'$ must vanish, in spite of the fact that $F(\tau')$ and $F(V')$ may have their maximum values.[1]

To find the distribution of x and y for small values of τ' and V' I have assumed that the distributions of these quantities, for uniform intervals $d\tau'$ and dV', are of such types that for values numerically less than certain limits the frequency is constant. That is, if signs were retained, the distributions would have flat tops and maxima around $\tau'=V'=0$. Even if the most frequent value of τ' or V' should differ slightly from zero because of uncertainties in the apex direction or from systematic errors, the effect on the distribution of the logarithms would be quite insignificant. In the case of the radial velocities I assume that the distribution is constant for $V' \leq a$. Below this limit the frequency of V' is thus given by the equation

$$F(V')dV'=\frac{n_1}{a}\,dV' ,$$

where n_1 is the number of values of V' numerically less than a.

From equations (5) we find, replacing T by V',

$$V'=47.38\ 10^{y} ,$$

and thus

$$dV'=\frac{47.38}{\text{mod}}\ 10^{y}\,dy .$$

Consequently

$$F_2(y)dy=\frac{n_1}{a}\,\frac{47.38}{\text{mod}}\ 10^{y}\,dy ,$$

and for any value of y less than $\log a-1.6756$,

$$\int_{y_1}^{y_2} F_2(y)dy=47.38\,\frac{n_1}{a}\,(10^{y_2}-10^{y_1}) . \qquad (6)$$

The n_1 values of y can thus be distributed in uniform intervals dy from $y=\log a-1.6756$ backward until the frequency falls to practically zero, when the table stops.

[1] In this connection a reference may be made to a note by Russell regarding the computation of geometrical mean parallaxes (*Publications of the Astronomical Society of the Pacific*, **32**, 243, 1920) and also a note by the writer (*ibid.*, p. 244).

A similar procedure is used for the τ-components. The distribution of the reduced components $\tau' = \tau \cdot 10^{0.2m}$ less than or equal to a certain value β is supposed to be constant, and we find as before

$$\int_{x_1}^{x_2} F_1(x)dx = \frac{n_2}{\beta}\,(10^{x_2} - 10^{x_1})\,, \tag{7}$$

where n_2 is the number of values of $\tau' \leq \beta$.

If the values of α and β can be so chosen that they are of the order of magnitude of the mean errors in V' and τ', the main effect of these errors is automatically corrected. A small effect is still left in the distributions $F_1(x)dx$ and $F_2(y)dy$, which could well be neglected. The additional corrections were, however, actually applied, and for completeness I shall here derive the necessary formulae.

In the general case denote true values by subscripts, the errors by ϵ, and the measured quantities by V. Thus

$$V_1 = V + \epsilon\,. \tag{8}$$

V_1 and ϵ can be regarded as mutually independent quantities. Denote the probabilities of true V, observed V, and errors by $F(V)dV$, $f(V)dV$, and $g(\epsilon)d\epsilon$, respectively. A little consideration shows that the following relation holds:

$$f(V)dV = dV \int_{-\infty}^{+\infty} F(V+\epsilon)g(\epsilon)d\epsilon\,. \tag{9}$$

Develop $F(V+\epsilon)$ in a power series retaining terms up to second order.

$$F(V+\epsilon) = F(V) + \frac{\epsilon dF}{dV} + \frac{\epsilon^2}{2}\frac{d^2F}{dV^2}\,. \tag{10}$$

It is here supposed that the convergence of this series is sufficiently rapid. Thus

$$f(V)dV = dV\left[F(V)\int_{-\infty}^{+\infty} g(\epsilon)d\epsilon + \frac{dF}{dV}\int_{-\infty}^{+\infty} \epsilon g(\epsilon) + \tfrac{1}{2}\frac{d^2F}{dV^2}\int_{-\infty}^{+\infty} \epsilon^2 g(\epsilon)d\epsilon\right]$$

or

$$f(V)dV = dV\left[F(V) + \bar{\epsilon}\,\frac{dF}{dV} + \frac{\bar{\epsilon}^2}{2}\frac{d^2F}{dV^2}\right]\,.$$

Replacing the derivatives of the function of true values with those of observed values, we have approximately

$$F(V)dV = dV\left[f(V) - \bar{\epsilon}\frac{df}{dV} - \frac{\bar{\epsilon}^2}{2}\frac{d^2f}{dV^2}\right]. \tag{11}$$

This procedure has often been used and is practically the same as that employed in *Contribution* No. 332.[1]

The numerical values of the derivatives may be found by either the graphical or the numerical method. In the present case, when the mean errors in τ' and V' are small compared with the dispersions themselves, we may with sufficient accuracy assume the distributions of V' and τ' to be normal error distributions. Denoting the dispersion in V' by σ_1 and the mean error in a determination of radial velocities by σ_2, we find, after taking into account that we are considering all values as positive,

$$\int_0^{V'} F(V')dV' - \int_0^{V'} f(V')dV' = \frac{N}{\sqrt{\pi}}\frac{\sigma_2^2}{\sigma_1^2} z e^{-z^2}, \tag{12}$$

where

$$z = \frac{V'}{\sqrt{2}\sigma_1}.$$

N is here the total number of stars in the group studied.

The limits in equation (12) are now chosen so as to conform to uniform intervals in $y = \log V' - 1.6756$, and the corrections thus derived are applied to the observed distribution $F_2(y)dy$.

The corrections to the distribution $F_1(x)dx$ are derived in exactly the same way from the mean error and the dispersion in τ'.

6. After the two distributions $F_1(x)dx$ and $F_2(y)dy$ have been found, the mean values of x and y might be formed, and the difference $\bar{x} - \bar{y}$ would give us the mean absolute magnitude immediately. This procedure would be identical with that proposed in 1918 in *Contribution* No. 144.[2] But we are not now interested in means; we want, if possible, to find the actual shape of the distribution-curve of absolute magnitudes.

[1] *Astrophysical Journal*, **65**, 238, 1927.

[2] *Ibid.*, **47**, 7, 1918.

If x and y were independent of each other, that is, if there were no correlation between them, the compounded distribution could be computed directly. But, since the angular motion is dependent on the linear tangential motion, there clearly must be a strong correlation between individual values of x and y. The independent variables on which the angular motion depends are linear tangential motion and distance, or, if the angular motions have been multiplied by $10^{0.2m}$, the independent variables are linear tangential motion and absolute magnitude, that is, y and M'. The dependence of linear motion upon absolute magnitude can be taken care of by using several distributions $F_2(y)dy$ for different limits in absolute magnitude.

As will be shown presently, the unknown distribution $F_3(M')dM'$ is determined by an integral equation ([15] below). The method used for solving it is an indirect one. At first a guess based on present knowledge is made of the distribution of absolute magnitudes, and then an expected distribution of x is computed from the assumed distribution of M' and the observed distribution of y. From the differences between the computed and the observed distributions of x, corrections to the assumed distribution of absolute magnitudes are derived and applied, and the process repeated until the observed distribution of reduced proper motion is well reproduced. The necessary formulae will now be derived.

We have the fundamental relation

$$x = M' + y\,, \tag{13}$$

where we may now assume M' and y to be independent variables. We can therefore compute the expected distribution $f_1(x)dx$ from the equation

$$\frac{1}{N_1} f_1(x)dx = \frac{dx}{N_2 N_3} \int_{-\infty}^{+\infty} F_2(x - M') f_3(M')dM'\,. \tag{14}$$

The approximate distribution of M' is here denoted by $f_3(M')dM'$, and N_1, N_2, N_3 are the total numbers of stars in the three distributions $F_1(x)dx$, $F_2(y)dy$, and $f_3(M')dM'$, respectively. Equation (14) is simply a combination of partial probabilities summed for

all values of M' and expressed as a function of x. Numerically it means that each of the numbers occurring in the sequence $F_2(y)dy$ is to be multiplied by each of the numbers in the sequence $f_3(M')dM'$ and the products grouped according to the values of $x=M'+y$, and then summed for each interval of x. It is practical to have the intervals dy and dM' equal, and then the interval dx will be the same as the other two. The numbers N_1, N_2, and N_3 can be chosen equal and the distributions reduced to this number beforehand.

A relation similar to equation (14) exists for the true distributions, $F_1(x)dx$ and $F_2(M')dM'$. Thus

$$\frac{1}{N_1} F_1(x)dx=\frac{dx}{N_2N_3}\int_{-\infty}^{+\infty} F_2(x-M')F_3(M')dM' . \tag{15}$$

This is the integral equation which must be solved numerically for $F_3(M')$. From (14) and (15) we find

$$[F_1(x)-f_1(x)]dx=\frac{N_1}{N_2N_3}\,dx\int_{-\infty}^{+\infty}[F_3(M')-f_3(M')]F_2(x-M')dM' . \tag{16}$$

Denote

$$[F_1(x)-f_1(x)]dx \text{ by } \Delta f_1(x)dx ,$$

and

$$[F_3(M')-f_3(M')]dM' \text{ by } \Delta f_3(M')dM' ,$$

and we have

$$\frac{N_1}{N_2N_3}\,dx\int_{-\infty}^{+\infty} F_2(x-M')\Delta f_3(M')dM'=\Delta f_1(x)dx . \tag{17}$$

We will now assume that $\Delta f_3(M')$ is approximately constant for a certain small range in M'. The limits used are M'_1, M'_2, M'_3, etc., and the corresponding means M'_{12}, M'_{23}, etc.

We have then

$$\frac{N_1}{N_2N_3}\,dx\left[\Delta f_3(M'_{12})\int_{M'_1}^{M'_2}F_2(x-M')dM'+\Delta f_3(M'_{23})\int_{M'_2}^{M'_3}F_2(x-M')dM'\right.$$
$$\left.+\text{etc.}\right]=\Delta f_1(x)dx .$$

But

$$\int_{M_1'}^{M_2'} F_2(x-M')dM' = \int_{x-M_1'}^{x-M_2'} F_2(y)dy\,,$$

and for other integrals we have similar equations. Hence

$$\left.\begin{aligned}\frac{N_1}{N_2N_3}\,dx\Big[\Delta f_3(M_{12}')\int_{x-M_1'}^{x-M_2'} F_2(y)dy+\Delta f_3(M_{23}')\int_{x-M_2'}^{x-M_3'} F_2(y)dy\\ +\text{etc.}\Big]=\Delta f_1(x)dx\,.\end{aligned}\right\} \tag{18}$$

For a range in x from x_1 to x_2, the mean of which is denoted by x_{12}, we may write the last equation in the form

$$\left.\begin{aligned}\frac{N_1}{N_2N_3}\,(x_2-x_1)\Big[\Delta f_3(M_{12}')\sum_{x_{12}-M_2'}^{x_{12}-M_1'} F_2(y)dy+\Delta f_3(M_{23}')\sum_{x_{12}-M_3'}^{x_{12}-M_2'} F_2(y)dy\\ +\text{etc.}\Big]=\sum_{x_1}^{x_2}\Delta f_1(x)dx\,.\end{aligned}\right\} \tag{19}$$

This equation may be regarded as an equation of condition for determining the unknown corrections $\Delta f_3(M')$ from the differences between observed and computed frequencies between the limits x_1 and x_2. For the next interval x_2 to x_3 a similar equation of condition may be obtained, and so on, until the whole range in x has been used.

The values of the absolute magnitude for which the corrections $\Delta f_3(M')$ are to be determined must not be taken too close together, for the least-squares solution will then become indeterminate. But by using overlapping intervals in x and in M' such that the means of x and M' are shifted the same amount, the corresponding y remains unchanged. The same coefficients in the equations of condition and in the normal equations can then be used for many determinations of $\Delta f_3(M')$ for as closely distributed values of M' as one may desire. The different solutions are, of course, not independent, but furnish corrections to the assumed distribution of absolute magnitude at points numerous enough to derive the corrected distribution with high precision and without excessive smoothing effect,

even though the true absolute-magnitude distribution changes rapidly.

It is clear that the different equations of condition (19) have greatly different weights, depending on the size of $F_1(x)dx$. It was found practical to apply to these equations weights inversely proportional to $F_1(x)dx$ after this distribution had been smoothed out.

When the improved absolute-magnitude distribution has been obtained, a new expected distribution of x is derived and compared with the observed distribution. If any systematic differences still exist, the process can be repeated. In general, two approximations have been found sufficient, even when the first approximation was very poor.

7. The integral equation (15) may not always have a definite solution. This depends upon the value of a certain determinant. If the value of this determinant is small, the solution becomes uncertain and may easily become unstable and indeterminate. The assumption of constant corrections $\Delta f_3(M')$ for a finite range in M' automatically eliminates very rapid oscillations in $F_3(M')$, which have no effect on the computed distribution of x and are thus indeterminate. The least-squares solution itself will tell if the intervals $M'_2-M'_1$, $M'_3-M'_2$, etc., have been chosen too small, for in this case the solution will become unstable. In this way a limit is set to the degree of resolution which can be obtained.

With slight modification the method can be applied to parallactic motions, and the necessary formulae and their application to K and M stars will be published in a future *Contribution*.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
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THE DISTRIBUTION OF ABSOLUTE MAGNITUDES AMONG K AND M STARS BRIGHTER THAN THE SIXTH APPARENT MAGNITUDE AS DETERMINED FROM PECULIAR VELOCITIES[1]

By GUSTAF STRÖMBERG

ABSTRACT

1. The *statistical method of determining the distribution of absolute magnitudes* from the reduced τ-*components and peculiar radial velocities* given in the preceding *Contribution* has been applied to the K and M stars in Boss's *Catalogue* brighter than *6 mag.* The stars were divided into three spectral groups, the number in each group being 659, 330, and 253, respectively.

Table I shows the distribution of $x = \log \tau + 0.2m$, as observed and also after correction for the effect of errors. It also shows the distribution of x computed from the adopted distribution of absolute magnitudes and of peculiar radial velocities V', and the differences between the observed and the computed distributions. The distributions of x are shown graphically in Figs. 1, 2, and 3.

The distribution of $y = \log V' - 1.6756$, as observed and after correction for the effect of errors in the radial velocities, is given in Table II for different absolute magnitudes.

The final distributions of absolute magnitudes are shown in Table IV and in Figs. 4, 5, and 6 with a class interval of 0.2 mag.

2. Among stars of spectral classes K0–K2 the absolute-magnitude distribution shows four distinct maxima, occurring at absolute magnitudes −1.8, +0.2, +2.9, and +6.7. The proportions of stars in the four groups are 9, 78, 12, and 1 per cent, respectively; these stars are designated by the terms "supergiants," "ordinary giants," "subgiants," and "dwarfs."

In the spectral interval K3–K9 three distinct frequency maxima occur, at −2.4, −0.1, and between 4 and 9, with relative proportions of 19, 78, and 3 per cent. These groups are identified as supergiants, ordinary giants, and dwarfs. No subgiants exist in this spectral interval.

Among the M stars only supergiants and ordinary giants appear. The frequency maxima occur at −1.6 and −0.1, and the relative proportions are 44 and 56 per cent.

3. Stable conditions in the stars and relatively rapid transitions from one stable state to another are suggested as the probable cause of the frequency maxima and minima.

The preceding *Contribution*, No. 395,[2] describes a method for determining as closely as possible the general shape of the distribution-curve of absolute magnitudes from the distribution of reduced τ-components and of peculiar radial velocities. This method has been applied to all the stars in Boss's *Catalogue*, excepting variables, equal to or brighter than Harvard apparent magnitude 5.99 and of spectral classes K–M. During the computation the method was

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 396.

[2] *Astrophysical Journal*, **71**, 163, 1930.

found to be more powerful than had been anticipated and capable of giving the distribution of absolute magnitude with high precision and great resolution, provided the distributions for the τ-components and the radial velocities are themselves well known.

The stars were divided into three spectral groups, Ko–K2, K3–K9, and Mo–M9, according to the Harvard classification. In some cases, however, Mount Wilson classifications were used, which in the mean give the type one or two subdivisions earlier than the Harvard classification. The numbers of stars in the three groups are 659, 330, and 253, or 53, 27, and 20 per cent, respectively.

The τ-components were taken from a typewritten copy prepared by prisoners in Holland under Kapteyn's direction. The second, third, and fourth columns of Table I give the distribution of $x = \log \tau + 0.2m$, as obtained directly from the original data, reduced to a total of one thousand stars. For convenience the interval is made uniform throughout and equal to 0.1. All the values of x less than -1.7 were combined in one group and then distributed according to the logarithmic scale. The distributions have also been corrected for the effect of errors in accordance with the formulae derived in the preceding *Contribution*. The mean error in a τ-component was assumed to be $\pm 0''.005$. The final distributions are given in the fifth, sixth, and seventh columns of Table I.

The computed distributions are given in the eighth, ninth, and tenth columns. These were derived from the last approximation for the absolute-magnitude distribution, which appears in Table IV, combined with the distribution of y in Table II. The last three columns of Table I give the differences between observed and computed distributions. The computed distribution-curves for the three spectral groups are shown graphically in Figures 1, 2, and 3. The observed values for the distribution are indicated by dots connected by straight lines.

The distributions of the linear velocities are in the second, third, and fourth columns of Table II between limits of $y = \log V' - 1.6756$, where V' is the peculiar radial velocity. To increase the data, the radial velocities for some stars fainter than sixth magnitude were included. This does not change the distribution of peculiar velocities, provided the fainter stars are not selected on the basis of large

proper motion, which would produce a certain increase in the mean linear motion. The numbers of stars used in the three groups were 447, 273, and 306, respectively.

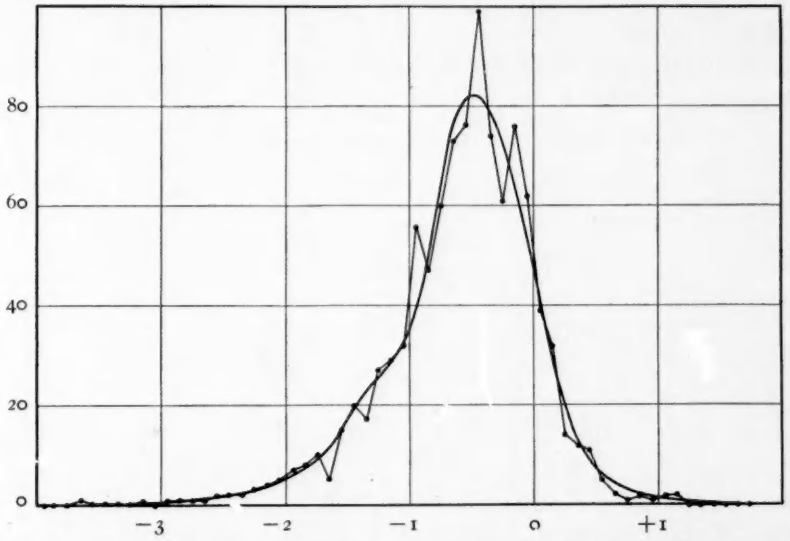

FIG. 1.—Distribution of $x = \log \pi + 0.2m$ for stars of spectral types K0 to K2. The dots joined by straight lines represent observed numbers within intervals $dx = 0.1$. The smooth curve is computed from the distributions of absolute magnitudes and peculiar radial velocities.

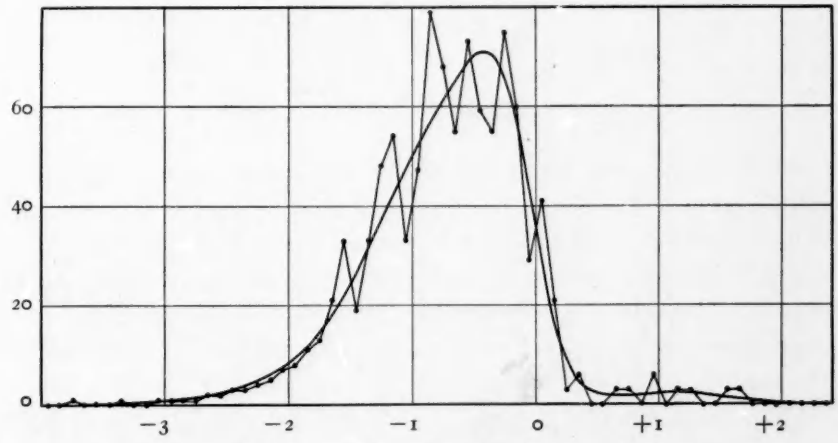

FIG. 2.—Same as Fig. 1 for stars of spectral types K3 to K9

The radial velocities, determined mainly at the Lick and Mount Wilson Observatories, were corrected for a constant solar motion of 20 km/sec. toward the apex $\alpha = 270°$, $\delta = +30°$. A moderate change in the solar motion does not appreciably influence the distribution of peculiar radial velocities.

The frequencies are reduced to a total of one thousand stars, those for $y < -1.2$, that is, for V' less than 3.0 km/sec., being grouped to-

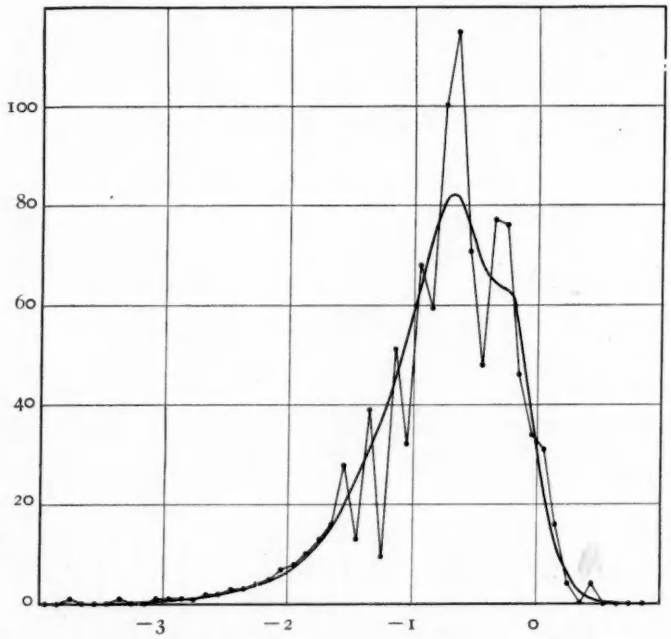

FIG. 3.—Same as Fig. 1 for stars of spectral type M

gether. The frequencies for the small values of V' were then found as before. The distributions were corrected for errors on the basis of a mean error of ± 3.0 km/sec. in the radial velocities, which is supposed to include systematic errors and errors in the assumed solar motion as well as accidental errors. The distributions of y were also smoothed graphically without losing the peculiarities in the distribution-curves.

The dependence of velocity distribution on absolute magnitude was derived from spectroscopic absolute magnitudes. This was done

TABLE I

$F(x)dx$

x	Observed Distribution			Observed Distribution Corrected for Errors			Computed Distribution			Observed *minus* Computed Distribution		
	K0–K2	K3–K9	M0–M9	K0–K2	K3–K9	M0–M9	K0–K2	K3–K9	M0–M9	K0–K2	K3–K9	M0–M9
−4.4				...	.....	.....	0.0	0.1	0.0	.....	− 0.1	0.0
−4.3				...	.....	.....	.0	.1	.0	.....	− .1	.0
−4.2				...	.....	.....	.0	.0	.0	.....	0.0	.0
−4.1				...	.....	.....	.1	.1	.1	− 0.1	− .1	− .1
−4.0				...	.....	.....	.0	.1	.1	.0	− .1	− .1
−3.9				...	.....	.....	.1	.1	.1	− .1	− .1	− .1
−3.8				...	1	1	.1	.2	.1	− .1	+ .8	+ .9
−3.7				1	0	0	.2	.3	.2	+ .8	− .3	− .2
−3.6				0	0	0	.2	.4	.3	− .2	− .4	− .3
−3.5				0	0	0	.2	.3	.3	− .2	− .3	− .3
−3.4				0	1	1	.3	.4	.4	− .3	+ .6	+ .6
−3.3				0	0	0	.3	.4	.5	− .3	− .4	− .5
−3.2	49	64	63	1	0	0	.5	.6	.6	+ .5	− .6	− .6
−3.1				0	1	1	.5	.9	.7	− .5	+ .1	+ .3
−3.0				1	1	1	.5	.9	.6	+ .5	+ .1	+ .4
−2.9				1	1	1	0.7	1.2	0.9	+ .3	− .2	+ .1
−2.8				1	1	1	1.1	1.6	1.3	− .1	− .6	− .3
−2.7				1	2	2	1.4	2.1	1.6	− .4	− .1	+ .4
−2.6				2	2	2	1.5	2.5	2.1	+ .5	− .5	− .1
−2.5				2	3	3	2.0	3.2	2.6	.0	− 0.2	+ .4
−2.4				2	3	3	2.6	4.0	3.2	− .6	− 1.0	− .2
−2.3				3	4	4	3.0	5.0	3.9	.0	− 1.0	+ .1
−2.2				4	5	5	3.8	6.1	4.9	+ .2	− 1.1	+ 0.1
−2.1				5	7	7	4.7	7.6	5.8	+ 0.3	− 0.6	+ 1.2
−2.0				7	8	8	6.0	9.6	7.3	+ 1.0	− 1.6	+ 0.7
−1.9				8	11	10	7.5	11.8	9.3	+ 0.5	− 0.8	+ 0.7
−1.8				10	13	13	9.4	14.6	12.0	+ 0.6	− 1.6	+ 1.0
−1.7	5	21	16	5	21	16	11.7	18.1	15.5	− 6.7	+ 2.9	+ 0.5
−1.6	15	33	28	15	33	28	15.3	22.1	19.9	− 0.3	+10.9	+ 8.1
−1.5	20	18	12	20	19	13	19.4	26.5	25.0	+ 0.6	− 7.5	−12.0
−1.4	17	33	39	17	33	39	22.9	31.8	31.0	− 5.9	+ 1.2	+ 8.0
−1.3	27	48	9	27	48	9	25.8	36.9	37.5	+ 1.2	+11.1	−28.5
−1.2	29	54	51	29	54	51	29.0	42.1	45.2	0.0	+11.9	+ 5.8
−1.1	32	33	32	32	33	32	32.7	47.4	54.1	− 0.7	−14.4	−22.1
−1.0	56	46	67	56	47	68	38.9	52.3	63.4	+17.1	− 5.3	+ 4.6
−0.9	47	79	59	47	79	59	49.8	56.6	72.9	− 2.8	+22.4	−13.9
−0.8	59	67	99	60	68	100	64.6	60.8	80.4	− 4.6	+ 7.2	+19.6
−0.7	73	55	115	73	55	115	76.5	64.9	81.8	− 3.5	− 9.9	+33.2
−0.6	76	73	71	76	73	71	81.4	68.4	75.7	− 5.4	+ 4.6	− 4.7
−0.5	99	58	47	99	59	48	81.7	70.8	68.0	+17.3	−11.8	−20.0
−0.4	74	55	77	74	55	77	78.5	70.2	64.7	− 4.5	−15.2	+12.3
−0.3	61	76	77	61	75	76	72.3	65.5	63.2	−11.3	+ 9.5	+12.8
−0.2	76	61	47	76	60	46	63.7	56.6	54.9	+12.3	+ 3.4	− 8.9
−0.1	62	30	35	62	29	34	52.9	44.0	39.9	+ 9.1	−15.0	− 5.9
0.0	38	42	32	38	41	31	41.3	29.2	24.5	− 3.3	+11.8	+ 6.5
+0.1	33	21	16	32	21	16	29.9	16.9	12.9	+ 2.1	+ 4.1	+ 3.1
+0.2	14	3	4	14	3	4	20.3	9.2	6.2	− 6.3	− 6.2	− 2.2
+0.3	12	6	0	12	6	0	13.4	5.1	2.7	− 1.4	+ 0.9	− 2.7
+0.4												

TABLE I—*Continued*

x	Observed Distribution			Observed Distribution Corrected for Errors			Computed Distribution			Observed *minus* Computed Distribution		
	Ko–K2	K3–K9	Mo–M9	Ko–K2	K3–K9	Mo–M9	Ko–K2	K3–K9	Mo–M9	Ko–K2	K3–K9	Mo–M9
+0.4	11	0	4	11	0	4	8.9	3.0	1.2	+ 2.1	− 3.0	+ 2.8
+0.5	5	0	0	5	0	0	6.4	2.2	0.4	− 1.4	− 2.2	− 0.4
+0.6	2	3	0	2	3	0	4.5	1.9	.1	− 2.5	+ 1.1	− .1
+0.7	1	3	0	1	3	0	3.2	1.8	0.0	− 2.2	+ 1.2	0.0
+0.8	2	0	0	2	0	0	2.3	2.0	.....	− 0.3	− 2.0	.....
+0.9	1	6	0	1	6	0	1.6	2.2	.....	− .6	+ 3.8	.....
+1.0	2	0	0	2	0	0	1.3	2.4	.....	+ .7	− 2.4	.....
+1.1	0	3	0	0	3	0	0.9	2.5	.....	− .9	+ 0.5	.....
+1.2	0	3	0	0	3	0	.8	2.4	.....	− .8	+ 0.6	.....
+1.3	0	0	0	0	0	0	.6	2.2	.....	− 0.6	− 2.2	.....
+1.4	2	0	0	2	0	0	.4	1.9	.....	+ 1.6	− 1.9	.....
+1.5	0	3	0	0	3	0	.2	1.6	.....	− 0.2	+ 1.4	.....
+1.6	0	3	0	0	3	0	.1	1.3	.....	− .1	+ 1.7	.....
+1.7	0	0	0	0	0	0	.1	1.1	.....	− .1	− 1.1	.....
+1.8	0	0	0	0	0	0	.0	0.7	.....	0.0	− 0.7	.....
+1.9	0	0	0	0	0	0	.0	.5	.....	.....	− .5	.....
+2.0	0	0	0	0	0	0	.0	.2	.....	.....	− .2	.....
+2.1	0	0	0	0	0	0	.0	.2	.....	.....	− .2	.....
+2.2	0	0	0	0	0	0	0.0	0.1	.....	.....	− 0.1	.....
+2.3												

by forming the means of the logarithms of the peculiar radial velocities for different absolute-magnitude groups. It was assumed that the shape of the distribution-curve of y for each spectral group was the same for different absolute magnitudes. By simply shifting the whole distribution-curve by an amount determined from the means of y it was thus possible to find the distribution for each absolute magnitude. The final distributions are given in Table II, the absolute magnitude at which the transition from one distribution to another takes place being given on top between the columns.

In the first approximation for the M stars it was assumed that the linear velocities are independent of the absolute magnitudes. It was then found that the absolute-magnitude distribution showed a separation into two distinct groups, and the mean velocity in each of these was found by using reduced proper motion as well as spectroscopic evidence as criteria for absolute magnitude. The data for reduced proper motion are given in *Contribution* No. 293.[1]

At the bottom of each column in Table II is given the arithmetical

[1] *Astrophysical Journal*, **61**, 363, 1925.

TABLE II

$F(y)dy$

y	K0–K2	K3–K9	M0–M9	K0–K2			K3–K9						M0–M9	
				−1.4	+5.0		−2.4	−2.0	−1.4	−0.6	+2		−0.9	
−4.0				1	.....	.....	1							
−3.9				0	.....	.....	0	1						
−3.8				0	.....	.....	0	0	1	.....	.....	.....	1	0
−3.7				0	1	1	0	0	0	1	.....	.....	0	0
−3.6				0	0	0	0	0	0	0	1	.....	0	1
−3.5				1	0	0	1	0	0	0	0	1	0	0
−3.4				0	0	0	0	1	0	0	0	0	0	0
−3.3				0	0	0	0	0	1	0	0	0	1	0
−3.2				1	1	1	1	0	0	1	0	0	0	0
−3.1				1	0	0	1	1	0	0	1	0	0	1
−3.0				1	0	0	1	1	1	0	0	1	1	0
−2.9				1	1	1	1	1	1	1	0	0	1	0
−2.8	123	121	111	2	1	1	2	1	1	1	1	0	1	1
−2.7				2	1	1	2	2	1	1	1	1	1	1
−2.6				2	1	1	2	2	2	1	1	1	1	1
−2.5				3	2	2	3	2	2	2	1	1	2	1
−2.4				4	2	2	4	3	2	2	2	1	2	1
−2.3				5	2	2	5	4	3	2	2	2	3	2
−2.2				7	3	3	6	5	4	3	2	2	4	3
−2.1				8	4	3	8	6	5	4	3	2	5	3
−2.0				10	5	4	10	8	6	5	4	3	6	4
−1.9				13	7	6	12	10	8	6	5	4	7	5
−1.8				16	8	7	17	12	10	8	6	5	8	6
−1.7				18	10	9	20	17	12	10	8	6	10	7
−1.6				22	13	11	23	20	17	12	10	8	12	9
−1.5				25	16	14	28	23	20	17	12	10	17	12
−1.4				29	18	17	34	28	23	20	17	12	22	16
−1.3				36	22	20	43	34	28	23	20	17	29	20
−1.2	22	33	42	58	25	23	53	43	34	28	23	20	37	26
−1.1														

TABLE II—*Continued*

y	K0–K2	K3–K9	M0–M9	K0–K2			K3–K9						M0–M9	
				−1.4		+5.0	−2.4	−2.0	−1.4	−0.6	+2		−0.9	
−1.1	29	47	43	84	29	27	63	53	43	34	28	23	46	34
−1.0	31	26	43	97	36	32	73	63	53	43	34	28	54	42
−0.9	63	44	62	101	58	42	84	73	63	53	43	34	64	50
−0.8	94	73	62	100	84	74	95	84	73	63	53	43	75	60
−0.7	89	60	65	92	97	92	99	95	84	73	63	53	86	70
−0.6	101	90	82	81	101	100	99	99	95	84	73	63	97	82
−0.5	96	92	92	65	100	101	87	99	99	95	84	73	111	92
−0.4	92	99	118	50	92	98	58	87	99	99	95	84	116	106
−0.3	65	99	114	33	81	86	32	58	87	99	99	95	92	116
−0.2	65	88	82	19	65	73	17	32	58	87	99	99	48	109
−0.1	72	59	39	8	50	58	8	17	32	58	87	99	24	64
0.0	25	33	23	3	33	41	4	8	17	32	58	87	10	32
+0.1	27	18	16	1	19	26	2	4	8	17	32	58	4	14
+0.2	2	11	3	0	8	14	1	2	4	8	17	32	2	5
+0.3	2	0	3	0	3	5	0	1	2	4	8	17	0	3
+0.4	0	4	0	0	1	2	0	0	1	2	4	8	0	1
+0.5	2	3	0	0	0	0	0	0	0	1	2	4	0	0
+0.6	0	0	0	0	0	0	0	0	0	0	1	2	0	0
+0.7	0	0	0	0	0	0	0	0	0	0	0	1	0	0
+0.8														
$\bar{y}$.....	−0.624	−0.600	−0.625	−0.921	−0.621	−0.570	−0.900	−0.800	−0.700	−0.600	−0.500	−0.400	−0.703	−0.558
$\underline{V}'$.....	11.25	11.89	11.24	5.69	11.25	12.76	5.96	7.51	9.45	11.90	14.98	18.86	9.40	13.12

mean $\bar{y}$ and the geometric mean $\underline{V}'$ of the peculiar radial velocities for the different types and absolute magnitudes. These means are connected by the relation

$$\bar{y} = \log \underline{V}' - 1.6756 .$$

TABLE III

$F(M)dM$

M	K0–K2	K3–K9	M0–M9
−6.0	0.3	− 4.7	.....
−5.5	.6	− 3.3	.....
−5.0	.8	− 2.9	− 2.6
−4.5	− .5	− 5.2	− 1.8
−4.0	− .4	8.4	0.0
−3.5	0.2	6.3	− 6.2
−3.0	3.1	11.7	−10.3
−2.5	6.8	35.7	20.0
−2.0	14.4	21.8	58.7
−1.5	14.3	26.9	80.4
−1.0	14.0	59.5	21.7
−0.5	42.0	93.7	61.4
0.0	127.6	112.6	84.8
+0.5	132.4	33.0	49.8
+1.0	8.3	− 4.6	22.3
+1.5	6.3	− 1.7	5.3
+2.0	6.9	− 1.7	1.0
+2.5	14.2	− 6.1	− 0.3
+3.0	14.4	− 1.5	− .6
+3.5	7.5	− 1.2	− .4
+4.0	3.7	− 0.8	− .2
+4.5	− 0.8	3.2	+ 0.5
+5.0	− 4.0	1.8	.....
+5.5	− 2.1	1.1	.....
+6.0	0.1	1.5	.....
+6.5	1.8	− 0.6	.....
+7.0	2.0	4.4	.....
+7.5	0.2	0.2	.....
+8.0	.2	− 1.0	.....
+8.5	.4	0.7	.....
+9.0	.2	1.3	.....
+9.5	− 0.1	0.7	.....

The assumed absolute-magnitude distributions used as first approximations in deriving the final distributions were in general taken from the spectroscopic data. The actual values used are of no importance, since several approximations were made before the representation of the observed distribution of x was regarded as satisfactory. The method is described in detail in the preceding *Contribution* and consists in combining the assumed absolute-magnitude distribu-

tion with the distribution of y to obtain a computed distribution $f(x)dx$, and then, from the differences between computed and observed distributions of x, in finding corrections to the assumed distribution of absolute magnitudes. The frequencies $F(M)dM$ thus derived from the final least-squares solution, referred to an interval of

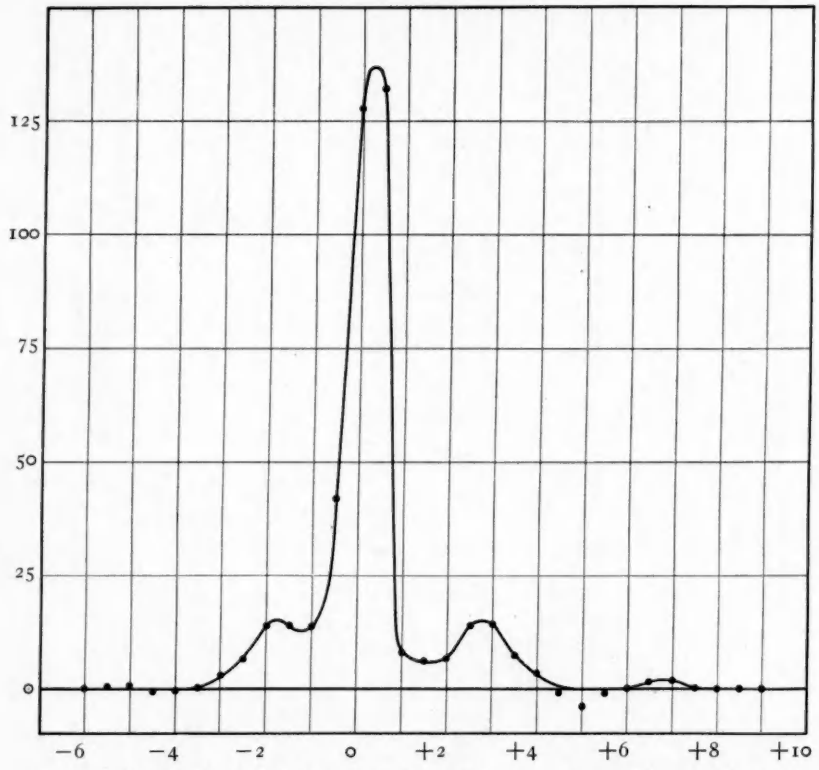

FIG. 4.—Distribution of absolute magnitudes for stars of spectral types K0 to K2. The points are computed from the distribution of reduced τ-components and peculiar radial velocities. Notice four frequency maxima.

0.2 mag. in dM, are given in Table III. The final distributions of absolute magnitude are shown graphically in Figures 4, 5, and 6, where the computed values are indicated by dots. Negative frequencies have been replaced by zero values, and the total numbers adjusted with the least amount of smoothing. The final distributions are shown by the curves, and the corresponding numbers appear in Table IV.

A study of the distribution of absolute magnitudes shown in Figures 4, 5, and 6 and in Table IV is of great interest. The K0–K2 stars show four distinct maxima, at −1.8, +0.2, +2.9, and +6.7 mags. The proportions of stars in the four groups are 9, 78, 12, and 1 per cent. The stars in these groups will here be designated "supergiants," "ordinary giants," "subgiants," and "dwarfs." The existence of supergiants was not unexpected, since spectroscopic criteria

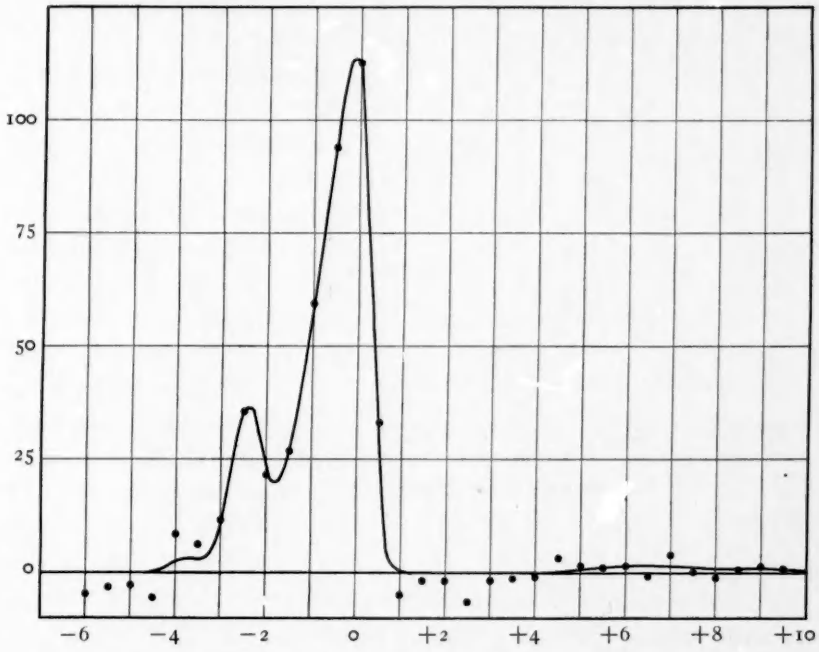

FIG. 5.—Absolute-magnitude distribution for stars of spectral types K3 to K9

have indicated their presence among stars of all types between F and M; but it is interesting to find that they are separated from the ordinary giants by a frequency minimum at −1.0. The most interesting feature of the distribution is the extremely rapid rise and fall around the maximum at +0.2. In spite of their great number, all the ordinary giants seem to be confined within an interval of 2 mags. The small range in M has also been indicated by the spectroscopic criteria, but additional evidence was desirable. The subgiants, as a distinct group of stars, were entirely unexpected. It has been known

that stars of an absolute magnitude intermediate between giants and dwarfs exist, but the presence of an intermediate group, separated on both sides by frequency minima, was not suspected. This group is probably rather prominent among the G stars, but does not seem to occur among other spectral classes. The dwarfs lie between absolute magnitudes 6 and 7.5, the fainter ones being excluded by the limitation in apparent magnitude.

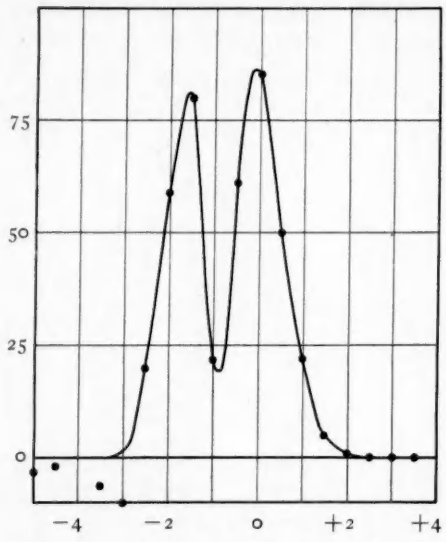

FIG. 6.—Absolute-magnitude distribution for stars of spectral type M.

Among stars of spectral class K3–K9 subgiants do not seem to occur. The maxima are at −2.4 for the supergiants, −0.1 for the ordinary giants, and between 5 and 9 for the dwarfs. The relative proportions are 19, 78, and 3 per cent, respectively, the supergiants thus being relatively more numerous than in the K0–K2 group. The minimum separating supergiants from the ordinary giants is now clearly marked. The rapid drop in frequency from absolute magnitude 0 to +1 is also here very pronounced, and no star seems to exist between absolute magnitudes +1 and +4.

The absolute magnitudes of the M stars fall into two distinct groups, the maximum frequencies occurring at −1.6 and −0.1. The relative proportions of supergiants and ordinary giants are 44 and 56 per cent. No subgiants or dwarfs seem to exist within the chosen limits of apparent magnitude. Because of the more evenly divided distribution, the minimum between the groups is now well marked.

The reality of these maxima and minima can, to a certain extent, be judged from the distributions of x in Figures 1, 2, and 3. The details in the absolute-magnitude distributions are, of course, much obscured by having been combined with the distributions of linear motion; but traces of the different maxima are left, in the form of

humps, on the curves. For the M stars the maximum in $F(x)dx$ corresponding to the supergiants is very pronounced in the observations, that corresponding to the ordinary giants less so. The whole

TABLE IV

$F(M)dM$

M	K0–K2	K3–K9	M0–M9	M	K0–K2	K3–K9	M0–M9
−4.2	0	2	0	+ 3.0	15	0	0
−4.0	0	3	0	+ 3.2	10	0	0
−3.8	0	3	0	+ 3.4	8	0	0
−3.6	0	3	0	+ 3.6	6	0	0
−3.4	1	4	0	+ 3.8	4	0	0
−3.2	2	8	0	+ 4.0	2	0	0
−3.0	4	17	2	+ 4.2	1	0	0
−2.8	5	27	9	+ 4.4	0	0	0
−2.6	7	36	20	+ 4.6	0	0	0
−2.4	10	36	37	+ 4.8	0	1	0
−2.2	13	27	52	+ 5.0	0	2	0
−2.0	15	20	67	+ 5.2	0	2	0
−1.8	15	21	78	+ 5.4	0	1	0
−1.6	14	27	80	+ 5.6	0	1	0
−1.4	13	38	55	+ 5.8	0	2	0
−1.2	13	51	31	+ 6.0	1	1	0
−1.0	15	67	19	+ 6.2	2	2	0
−0.8	19	82	30	+ 6.4	2	1	0
−0.6	39	95	61	+ 6.6	2	2	0
−0.4	74	107	82	+ 6.8	2	2	0
−0.2	111	113	86	+ 7.0	2	1	0
−0.0	135	94	79	+ 7.2	1	2	0
+0.2	137	59	65	+ 7.4	0	1	0
+0.4	129	25	50	+ 7.6	0	1	0
+0.6	70	4	36	+ 7.8	0	1	0
+0.8	11	0	26	+ 8.0	0	1	0
+1.0	7	0	17	+ 8.2	0	1	0
+1.2	6	0	9	+ 8.4	0	1	0
+1.4	5	0	5	+ 8.6	0	1	0
+1.6	5	0	3	+ 8.8	0	1	0
+1.8	6	0	1	+ 9.0	0	1	0
+2.0	8	0	0	+ 9.2	0	1	0
+2.2	11	0	0	+ 9.4	0	0	0
+2.4	14	0	0	+ 9.6	0	1	0
+2.6	15	0	0	+ 9.8	0	0	0
+2.8	15	0	0	+10.0			
+3.0							

run is, however, in this case rather erratic, because of the small number of stars and the small intervals used.

It is clear that the process of deriving the absolute-magnitude distribution from the distribution of reduced τ-components implies that humps of small amplitude and large extension in $F_1(x)dx$ determine humps of large amplitude and small extension in the absolute-

magnitude distribution. Since the reality of the small humps in $F_1(x)dx$ is difficult to prove or disprove, the demonstration of the existence of the maxima and minima in the absolute-magnitude distribution must be based principally on the smooth running of the curves and the general consistency of the results for different spectral types. Additional evidence may be forthcoming when the results from parallactic motions become available.

The effect of systematic errors in the proper motions was investigated by applying Raymond's corrections[1] to the proper motions in declination and computing the effect on the τ-components. It was found that the influence is entirely negligible and may be regarded as taken into account by the correction for accidental errors.

A comparison with the spectroscopic absolute magnitudes shows a very good agreement, except for the supergiants. There are several reasons for this discrepancy. One is that the stars in the present investigation are distributed all over the sky, whereas the spectroscopic data are limited to stars north of $-25°$ declination. This limitation is significant since there is reason to believe that the supergiants are particularly numerous in the Southern Hemisphere. This selective effect might be expected to be most pronounced among the M stars which, according to the present study, include a large percentage of supergiants.

It has been known for some time that the mean error in the spectroscopic absolute magnitudes is considerably larger for the supergiants than for stars in general. For the M stars the writer found[2] the probable errors for the two classes of stars to be 0.63 and 0.45, respectively. The reason for this may be partly a smaller coefficient of correlation between line intensity and absolute magnitude and partly the fact that different reduction-curves are used for the two classes of stars, and it may be uncertain which reduction-curve should be used for stars with intermediate line intensities. It must be noted that the minimum separating the two classes is extremely narrow, and even a small extra dispersion may obliterate it.

The existence of a numerous group of supergiant M stars has been

[1] *Astronomical Journal*, **36**, 129, 1926.

[2] *Mt. Wilson Contr.*, No. 327; *Astrophysical Journal*, **65**, 108, 1927.

foreshadowed by certain discoveries. Adams, Joy, and Humason[1] first called attention to the fact that the M stars brighter than absolute magnitude −1.0 show a pronounced galactic concentration; and Luyten,[2] using spectroscopic absolute magnitudes, has shown that the giant M stars in galactic latitudes less than 20° are 0.7 mag. brighter than those in higher latitudes. Finally, as mentioned above, I have found a large difference in peculiar and solar motions for two groups of giant M stars, selected on the basis of $m+5 \log \mu$. All these phenomena are most easily explained by the existence of two distinct groups of different absolute magnitudes and different galactic concentrations.

A particular advantage of the present method of determining the distribution of absolute magnitudes is that the distribution is determined directly, without introducing extra spread or losing inherent spread in the distribution of true absolute magnitudes. This is in contradistinction to methods based on the existence of a correlation between physical characteristics of a star and its intrinsic brightness, which, while giving individual absolute magnitudes, may not give the distribution of absolute magnitudes as accurately as the present statistical method. With such methods minor frequency maxima and minima may be blotted out by the extra dispersion introduced by the circumstance that we are dealing with correlations and not with direct observations of absolute magnitude. On the other hand, the methods which give individual absolute magnitudes may be applied to faint and distant stars for which adequate proper-motion data may never be available.

It has often been suggested that the existence of giants and dwarfs is due to the fact that certain absolute magnitudes are connected with stable conditions in the stars. The existence of additional frequency maxima, as indicated by the present study, can similarly be explained, since several stable states with different degrees of stability can well be imagined, the degree being represented by the relative abundance and the duration in any given state. Transitions between different states may occur, but the stars do not remain long

[1] *Mt. Wilson Contr.*, No. 319; *Astrophysical Journal*, **64**, 225, 1926.

[2] *Proceedings of the National Academy of Sciences*, **14**, 488, 1928.

in the intermediate states. The proof of the existence of new frequency maxima and minima may thus be of importance for the solution of the problem of stellar evolution.

It is to be noted, however, that in order to derive the relative duration of different states in stellar evolution the frequencies here derived must first be converted into frequencies per unit of volume. Since the present investigation refers only to stars brighter than sixth magnitude it is necessary first to find the distribution in absolute magnitude among stars of fainter apparent magnitudes.

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THE CONTOURS OF HYDROGEN LINES IN STELLAR SPECTRA

By C. T. ELVEY

ABSTRACT

The *contours of the hydrogen lines Hβ* and *Hγ* have been determined for sixty-four stars, mostly of spectral types earlier than Fo. The residual intensities and the widths of the lines of the early type stars are greater than for the lines of the same total intensity in later types. This is interpreted as the result of the Stark effect.

The *excess widths* of the lines are correlated with the *absolute magnitudes* of the stars. Another correlation with the absolute magnitudes is the ratio of the width of the line to its depth. The correlations are used to determine the absolute magnitudes of several stars.

The effective *numbers of atoms* producing the absorption lines *are determined* and compared with Milne's application of the theory of thermal ionization to a stellar atmosphere. The observations are *not in accordance with the theory in its present form.*

The observations show an *absolute-magnitude effect* for stars at temperatures higher than that of maximum, which is *in contradiction to the "null effect"* of Milne.

In a note added to the paper the suggestion of Gaunt that the coefficient of absorption varies with the temperature as $P/T^{3/2}$ instead of as $P/T^{9/2}$, which is used in Milne's formula, gives a theoretical curve for the variation of the number of atoms with the temperature which is in much better agreement with the observations.

There are probably three agencies tending to produce widening of the hydrogen absorption lines in stellar spectra. These causes are: (1) the abundance of atoms, (2) axial rotation of the star, and (3) the Stark effect. The abundance-broadening of spectral lines has been discussed by J. Q. Stewart,[1] A. Unsöld,[2] E. A. Milne,[3] W. S. Adams and H. N. Russell,[4] and others.

For the abundance-broadening we shall use the formula given by Unsöld for the contour of the absorption line, and the application of the theory of thermal ionization as given by Milne.

The effect of axial rotation upon the contour of a spectral line has been discussed by G. Shajn and O. Struve[5] and others. The effect is most pronounced for narrow, deep lines which become wide, shallow lines when the rotation is very rapid. The resulting contours resemble the "dish-shaped" ones obtained by the writer[6] for the helium

[1] *Astrophysical Journal*, **59**, 30, 1924. [2] *Zeitschrift für Physik*, **46**, 765, 1928.

[3] *Monthly Notices*, **89**, 3, 1928; *Philosophical Transactions of the Royal Society of London*, A, **228**, 421, 1929.

[4] *Astrophysical Journal*, **68**, 9, 1928. [5] *Monthly Notices*, **89**, 222, 1929.

[6] *Astrophysical Journal*, **70**, 141, 1929.

lines in a few stars, and consequently attributed to rotation. However, in the case of the helium lines there may be some uncertainty due to the fact that, next to hydrogen, helium atoms are the most influenced by the Stark effect. A heavier atom, such as magnesium,[1] is less affected by electrical fields and is better to show the effect of rotation.

O. Struve[2] and others have discussed the influence of mol-electric fields on the lines in stellar spectra. The contours of the helium lines[3] in stellar spectra show definite evidence of the Stark effect. The pressure effect of certain lines in stellar spectra has been shown by Struve[4] to be due to the broadening of the lines in an electric field. Miss Emma Williams[5] has shown that the correlation of the width and depth of the hydrogen lines in stellar spectra with the absolute magnitude of the stars is very probably the result of the Stark effect.

The broadening of the lines of hydrogen due to the Stark effect is larger than for any other element, and hence one would expect it to be easily recognized. The stars which are rotating rapidly can be sorted out by an inspection of the contours of lines which are very little affected by mol-electric broadening. It is only the largest rotations that affect the contours of lines as intense as those of hydrogen. As has been shown for the supergiant star 7 ϵ Aurigae,[6] the contour of $H\beta$ is very well represented by Unsöld's theoretical curve for abundance-broadening when the density is sufficiently low to eliminate the broadening due to Stark effect and collisions. For a group of non-rotating stars, the deviations of the contours of the hydrogen lines from the theoretical ones may be considered to be due to the Stark effect.

The material used in this discussion is represented by the spectrograms of sixty-four stars, mostly of spectral classes B and A, taken with the Bruce spectrograph of the Yerkes Observatory. The dispersion of the spectrograph is that of three prisms, with a resulting

[1] The Rotation of Stars and the Contours of Mg^+ 4481. Elvey, *Astrophysical Journal*, **71**, 221, 1930.

[2] *Ibid.*, **69**, 163, 1929. [3] *Elvey, ibid.*, 237, 1929; **70**, 141, 1929. [4] *Ibid.*, **70**, 85, 1929.

[5] *Proceedings of the National Academy of Sciences*, **15**, 832, 1929.

[6] Struve and Elvey, *Nature*, **125**, 308, 1930, *Astrophysical Journal*, **71**, 136, 1930.

scale of 16 A/mm at $H\beta$ and 8 A/mm at $H\gamma$. The spectra were photographed on Eastman-40 plates, calibrated with a tube sensitometer, and developed with Eastman D-11 for five minutes. The spectrograms were analyzed with a registering microphotometer to obtain the contours of the spectral lines.

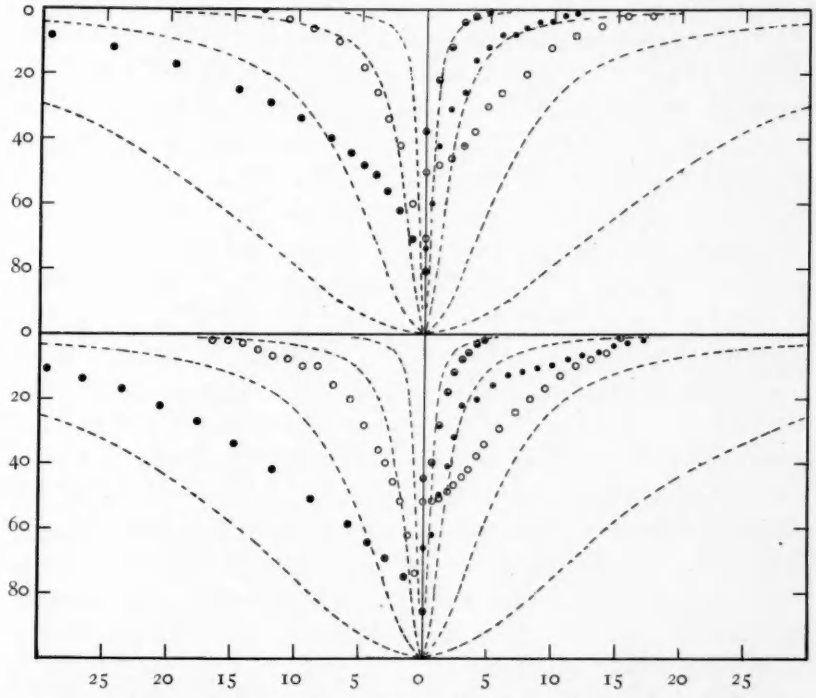

FIG. 1.—Contours of hydrogen lines, upper, $H\beta$; lower, $H\gamma$. The ordinates are percentages of absorption, the abscissae are angstrom units, and the dotted curves are Unsöld-contours for NHf equal to 10^{17}, 10^{18}, 10^{19}, and 10^{20}. Right side: circles, 32 α Leonis, B8; half-filled circles, 19 β Orionis, B8p; dots, 2 β Canis Majoris, B1. Left side: circles, 30 η Leonis, Aop; dots, 9 α Canis Majoris, A0.

A number of investigators[1] have published contours of the hydrogen lines, and hence no attempt will be made to give in detail the contours of the lines obtained in this investigation. A few samples are shown in Figure 1, as well as the computed contours according

[1] K. F. Bottlinger, *Astronomische Nachrichten*, **195**, 117, 1913; A. Unsöld, *Zeitschrift für Physik*, **46**, 765, 1927; T. Dunham, Jr., *Harvard Observatory Bulletin*, No. 858, 1928; Misses Payne and Williams, *Monthly Notices*, **89**, 526, 1929; Elvey, *Astrophysical Journal*, **68**, 145, 1928.

to the formula of Unsöld. The dotted curves are for NHf, the effective number of atoms, equal to 10^{17}, 10^{18}, 10^{19}, and 10^{20}. The abscissae of the diagrams are $(\lambda-\lambda_0)$ in angstroms, and ordinates are percentages of absorption of the continuous spectrum. In this figure are the contours of $H\beta$ and $H\gamma$ for five stars. The star 2 β Canis Majoris, of spectral class $B1$, has rather deep lines for a class B star and less tendency to have extensive wings in comparison to its depth. The star 32 α Leonis, B8, has lines that are distinctly rounded at the bottom, which may be interpreted as an effect of rotation. The hydrogen lines are the only ones that can be seen on the three-prism spectrograms.[1] One may compare its lines with those of the supergiant star 19 β Orionis, of spectral class B8p and of estimated absolute magnitude -5.8. The lines in this star are not particularly deep, but are quite narrow, the width being near enough to the limit to question the accuracy of the contour. Also, 32 α Leonis may be compared with 30 η Leonis, a star of type Aop which has lines of about the same total area. It will be seen, however, that the contours of the lines in this star conform much more closely to the theoretical ones. This star is probably a giant, as is indicated below. The other star whose contours have been plotted in Figure 1 is 9 α Canis Majoris. Its spectrum is typical of class Ao, and its absolute magnitude is 1.3. It has the most intense lines of the stars considered, the value of NHf being greater than 10^{19}. Likewise, the observed half-widths of its lines at 10 per cent absorption are the largest, 27.0 A for $H\beta$ and 29.8 A for $H\gamma$.

In order to show some of the differences of the fainter lines, the contours of $H\beta$ for five stars have been plotted on an enlarged scale in Figure 2. The abscissae are angstrom units, and the ordinates are percentages of absorption of the continuous spectrum. The large circles are the mean observed points on the contour of $H\beta$ in the spectrum of 7 ϵ Aurigae, and the dotted line is the theoretical contour for NHf equal to 2.5×10^{17}, which is the best fit to the observations. This contour has been discussed elsewhere.[2] We may compare the contours of the $H\beta$ line of some of the early type stars with this

[1] Some of the stronger metallic lines can be seen on one-prism spectra, but are faint and diffuse.

[2] Struve and Elvey, *Astrophysical Journal*, **71**, 136, 1930.

one which so closely fits the curves for abundance-broadening. A line of about the same total area is $H\beta$ in the spectrum of 2 β Canis Majoris, already shown in the preceding figure but plotted on the enlarged scale for comparison. This contour is represented by the half-filled circles on the diagram. A more typical star of the early classes is perhaps 88 γ Pegasi, type B2, whose $H\beta$ line is shown by the small circles. As may be seen, this contour deviates very markedly from the curves for abundance-broadening. Even greater deviations are shown by the contours of the lines in 46 ϵ Orionis, Bo (represented by the dots), and 39 λ Orionis, O8 (crosses). The Mg^+ line λ 4481 for these two stars indicates rotation. The contours of the lines in 39 λ Orionis are very much flattened, approaching the "dish-shaped" contours noted in the contours of the helium lines. Below, it is noted that the value of $I_o/(\lambda-\lambda_o)$ is much smaller than would be expected and probably indicates also the effect of rotation.

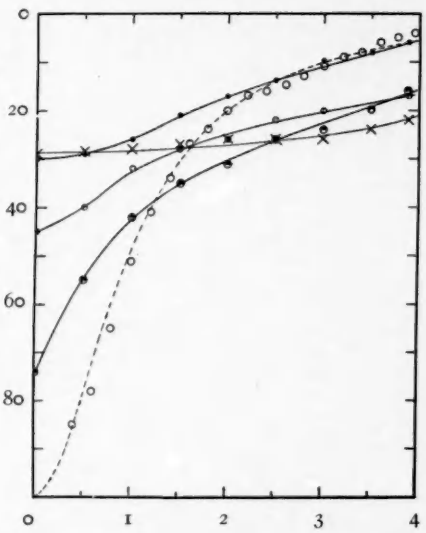

FIG. 2.—Contours of $H\beta$. The ordinates are percentages of absorption and the abscissae are angstrom units. The dotted curve is an Unsöld-contour for NHf equal to 2.5×10^{17}, which represents the observations for 7 ϵ Aurigae (large circles) very closely. Other observations deviate markedly from theoretical contours. Half-filled circles, 2 β Canis Majoris, B1; small circles 88 γ Pegasi, B2; dots, 46 ϵ Orionis, Bo; and crosses, 39 λ Orionis, O8.

It will be seen from the foregoing contours that the central absorption in the line is less in the case of the early type stars than for the lines of equal area in the later types. This is seen also by the values of I_o in Table I. The contour of $H\beta$ in 7 ϵ Aurigae shows that it is possible for the instrument to record much sharper lines than is indicated for the early class B stars. The deviations of the contours from the theoretical ones given by the abundance-broadening

TABLE I

No.	1 Star	2 Spec. Type	3 M	4 Total Absorption $H\beta$	5 Total Absorption $H\gamma$	6 log NHf $H\beta$	7 log NHf $H\gamma$	8 Δ log $\frac{NHf}{H\gamma - H\beta}$
1.....	22 λ Cep.	O6	−1.3	1.98	2.92	17.04	17.46	+0.42
2.....	30 τ CMa.	O8	.....	3.06	3.64	17.41	17.65	.24
3.....	44 ι Ori.	O8	−2.4	3.16	3.34	17.46	17.58	.12
4.....	39 λ Ori.	O8	−2.1	4.74	.....	17.78	.....	.....
5.....	10 Lac.	O9	.....	3.36	5.96	17.48	18.08	.60
6.....	46 ε Ori.	Bo	−4.0	1.88	1.90	17.00	17.12	+ .12
7.....	47 ρ Leo.	Bop	−2	2.66	1.90	17.29	17.12	− .17
8.....	2 β CMa.	B1	−2.6	3.66	5.90	17.56	18.06	+ .50
9.....	21 ε CMa.	B1	−3.2	3.36	.....	17.48	.....	.....
10.....	44 ζ Per.	B1	−3.2	2.18	2.50	17.12	17.33	.21
11.....	88 γ Peg.	B2	−1.8	3.36	4.40	17.48	17.81	.33
12.....	24 γ Ori.	B2	−2.0	4.16	4.24	17.67	17.78	.11
13.....	45 ε Cas.	B3	−1.0	5.72	6.80	17.95	18.19	.24
14.....	17 ζ Cas.	B3	−2.3	4.16	5.78	17.67	18.05	.38
15.....	85 ι Her.	B3	−1.8	6.52	6.56	18.06	18.16	.10
16.....	102 Her.	B3	−2.3	3.96	6.08	17.63	18.09	.45
17.....	20 η Lyr.	B3	−2.3	6.72	6.44	18.09	18.14	.05
18.....	3 π^4 Ori.	B3	−2.3	3.76	6.32	17.59	18.13	.54
19.....	22 τ Her.	B5	.....	7.70	8.76	18.21	18.42	.21
20.....	22 ζ Dra.	B5	−0.8	7.40	9.30	18.17	18.47	.30
21.....	2 ε Del.	B5	− .8	7.50	8.94	18.19	18.44	.25
22.....	19 q Tau.	B5	− .5	7.40	10.62	18.17	18.59	.42
23.....	26 β Per.	B8	−0.7	7.90	8.94	18.23	18.44	.21
24.....	4 γ Crv.	B8	.....	9.48	10.98	18.39	18.62	.23
25.....	32 α Leo.	B8	0.0	6.32	9.52	18.04	18.49	.45
26.....	19 β Ori.	B8p	−5.8	1.18	1.78	16.61	17.06	.45
27.....	16 λ Aql.	B9	+1.5	9.78	15.50	18.42	18.93	.51
28.....	3 α Lyr.	Ao	+0.6	18.18	21.18	18.97	19.22	.25
29.....	9 α CMa.	Ao	+1.3	18.98	23.60	19.01	19.32	.31
30.....	65 θ Aql.	Ao	−0.2	9.28	13.36	18.37	18.79	.42
31.....	66 α^2 Gem.	Ao	+1.4	13.52	17.90	18.70	19.06	+ .36
32.....	66 α^1 Gem.	Ao	+2.2	13.42	11.58	18.69	18.67	− .02
33.....	24 γ Gem.	Ao	+0.1	12.54	16.82	18.63	19.01	+ .38
34.....	5 α Cr.B.	Ao	+ .4	16.60	16.34	18.64	18.98	.34
35.....	48 β UMa.	Ao	+ .8	15.60	17.64	18.83	19.05	.22
36.....	30 η Leo.	Aop	.....	5.64	7.04	17.94	18.22	.28
37.....	34 β Aur.	Aop	− .2	15.80	16.92	18.84	19.01	.17
38.....	77 ε UMa.	Aop	−0.2	11.66	14.68	18.57	18.88	.31
39.....	12 α^2 CVn.	Aop	−1.2	10.68	10.12	18.55	18.55	± .00
40.....	69 δ UMa.	A2	+1.7	12.64	14.90	18.64	18.90	+ .26
41.....	94 β Leo.	A2	+2.2	17.20	17.34	18.92	19.04	.12
42.....	13 γ Umi.	A2	+1.0	9.88	12.82	18.42	18.76	.34
43.....	65 δ Her.	A2	+0.5	13.04	13.48	18.67	18.80	.13
44.....	ζ UMa (Seq.).	A2	+1.9	14.32	16.10	18.75	18.97	.22
45.....	79 ζ Uma.	A2p	+0.3	16.80	21.48	18.90	19.23	.33
46.....	50 α Cyg.	A2p	−5.2	2.76	3.16	17.32	17.53	.21
47.....	24 α PsA.	A3	+2.0	18.18	18.08	18.97	19.07	.10
48.....	68 δ Leo.	A3	+2.0	9.58	14.80	18.40	18.89	+ .49
49.....	111 Her.	A3	+3.0	16.30	11.68	18.87	18.67	− .20
50.....	4 β Tri.	A5	+1.1	12.34	16.10	18.62	18.97	+0.35

TABLE I *A*

No.	9	10	11 *Hβ*	12	13	14	15	16 *Hγ*	17	18
	$(\lambda-\lambda_0)$ Calc.	$(\lambda-\lambda_0)$ (Obs.)	(O−C)	I_0	$\frac{I_0}{(\lambda-\lambda_0)}$	$(\lambda-\lambda_0)$ Calc.	$(\lambda-\lambda_0)$ (Obs.)	(O−C)	I_0	$\frac{I_0}{(\lambda-\lambda_0)}$
1	2.0	3.9	1.9	31	8.0	2.8	4.4	1.6	50	11.4
2	3.0	5.4	2.4	40	7.4	3.5	5.3	1.8	49	9.2
3	3.2	4.9	1.7	39	8.0	3.3	5.5	2.2	38	6.9
4	4.6	9.3	4.7	29	3.1	.....	.....	.....	.....	.....
5	3.1	4.4	1.3	64	14.5	5.8	9.5	3.7	69	7.2
6	1.9	2.9	1.0	30	10.3	1.9	3.0	1.1	37	12.3
7	2.6	4.3	1.7	40	9.3	1.9	2.7	0.8	42	15.6
8	3.6	5.4	1.8	74	13.7	6.0	9.7	3.7	66	6.8
9	3.3	4.6	1.3	64	13.9	.....	.....	.....	.....	.....
10	2.2	3.2	1.0	46	14.4	2.4	4.0	1.6	60	15.0
11	3.3	5.6	2.3	45	8.0	4.2	7.7	3.5	58	7.5
12	4.1	6.4	2.3	55	8.6	4.1	6.8	2.7	53	7.8
13	5.7	8.1	2.4	68	8.4	6.7	10.3	3.6	72	7.0
14	4.1	6.1	2.0	58	9.5	5.6	9.2	3.6	68	7.4
15	6.4	8.6	2.2	65	7.5	6.4	8.2	1.8	>60	.....
16	3.9	5.9	2.0	56	9.5	5.8	8.8	3.0	68	7.7
17	6.5	7.8	1.3	>75	> 9.6	6.3	9.2	2.9	>50	.....
18	3.7	6.4	2.7	55	8.6	6.1	8.8	2.7	68	7.7
19	7.5	11.2	3.7	52	4.6	8.6	12.1	3.5	>57	.....
20	7.1	10.8	3.7	64	5.9	9.2	13.3	4.1	66	5.0
21	7.5	12.2	4.7	>56	> 4.6	8.8	12.4	3.6	.....	.....
22	7.1	10.8	3.7	62	5.7	10.6	13.9	3.3	74	5.3
23	7.8	12.2	4.4	62	5.1	8.8	14.8	6.0	64	4.3
24	9.4	11.8	2.4	68	5.8	10.8	14.5	3.7	75	5.2
25	6.2	10.0	3.8	45	4.5	9.2	14.5	5.3	55	3.8
26	1.2	2.1	0.9	43	20.5	1.8	2.8	1.0	55	19.9
27	9.6	14.2	5.6	72	5.1	15.3	16.8	1.5	>55	.....
28	17.9	26.5	8.6	78	2.9	21.6	29.0	7.4	84	2.9
29	18.8	27.0	8.2	81	3.0	24.2	29.8	5.6	86	2.9
30	9.0	13.5	4.5	62	4.6	13.3	15.5	2.2	>78	.....
31	13.3	18.2	4.9	>72	> 3.9	18.0	22.7	4.7	87	3.8
32	13.0	20.6	7.6	>60	> 2.9	11.6	18.3	6.7	55	3.0
33	12.4	17.2	4.8	83	4.8	16.6	21.3	4.7	>75	.....
34	12.4	22.1	9.7	65	2.9	16.4	21.9	5.5	67	3.1
35	15.6	20.6	5.0	83	4.0	17.6	23.6	6.0	77	3.3
36	5.5	6.9	1.4	71	10.3	6.8	9.2	2.4	>76	>8.3
37	15.6	20.8	5.2	61	2.9	16.6	20.6	4.0	71	3.4
38	11.3	16.2	4.9	70	4.3	14.6	19.2	4.6	82	4.3
39	11.3	15.2	3.9	79	5.2	10.1	14.8	4.7	78	5.3
40	12.4	17.6	5.2	66	3.8	15.0	21.2	6.2	72	3.4
41	17.2	25.0	7.8	69	2.8	17.6	23.9	6.3	73	3.1
42	9.6	13.7	4.1	75	5.5	12.8	16.0	3.2	80	5.0
43	13.0	20.1	7.1	50	2.5	13.4	19.5	6.1	57	2.9
44	14.2	22.1	7.9	77	3.8	16.0	25.1	9.1	63	2.5
45	16.8	20.4	3.6	75	3.7	22.1	28.3	6.2	75	2.6
46	2.7	4.2	1.5	65	15.5	3.1	4.0	0.9	76	19.0
47	17.9	23.0	5.1	78	3.4	18.4	23.0	4.6	73	3.2
48	9.4	14.2	4.8	66	4.6	14.6	19.8	5.2	74	3.7
49	16.4	27.5	11.1	55	2.0	11.6	18.3	6.7	52	2.8
50	12.1	18.6	6.5	80	4.3	16.0	20.6	4.6	>70	.....

TABLE I—*Continued*

No.	1 Star	2 Spec. Type	3 M	4 Total Absorption $H\beta$	5 Total Absorption $H\gamma$	6 log NHf $H\beta$	7 log NHf $H\gamma$	8 $\Delta \log \frac{NHf}{H\gamma - H\beta}$
51.....	5 α Cep.	A5	+2.2	7.70	13.12	18.21	18.78	0.57
52.....	53 α Aql.	A5	+2.4	13.72	13.12	18.71	18.78	+ .07
53.....	9 ι UMa.	A5	+2.3	13.34	8.34	18.69	18.37	− .32
54.....	55 δ Gem.	Fo	+2.6	4.24	7.04	17.69	18.23	+ .54
55.....	29 γ Vir. S.F.	Fo	+2.9	7.50	3.76	18.19	17.68	− .51
56.....	29 γ Vir. N.P.	Fo	+2.9	8.70	7.10	18.32	18.23	− .09
57.....	30 δ Aql.	Fo	+2.2	9.88	8.52	18.42	18.39	− .03
58.....	41 Cyg.	F5	−1.1	6.52	9.42	18.06	18.48	+ .42
59.....	33 α Per.	F5	−2.2	4.94	5.12	17.82	17.94	+ .12
60.....	7 ε Aur.	gF5	−4.0	.....	.....	17.40	.....	.....
61.....	10 α CMi.	dF5	+3.5	2.56	2.20	17.26	17.22	− .04
62.....	13 α Aur.	Go	−0.8	1.78	.....	16.96	.....	.....
63.....	Sun	Go	+4.8	0.98	1.20	16.46	16.74	+0.28
64.....	87 α Tau.	K5	−0.1	0.98	.....	16.46	.....	.....
65.....	67 Oph.	B5p	−5.0	.....	4.98	.....	17.92	.....
66.....	70 Oph.	Ko	+5.5	2.42	.....	17.21	.....	.....
67.....	α Boo.	Ko	−0.3	1.48	.....	16.80	.....	.....

indicate that one of the other two mechanisms responsible for broadening of spectral lines is at work in the early types. The effect of rotation can be eliminated by referring to lines little affected by the Stark effect. The contours of the Mg^+ λ 4481 line show that the majority of the early type stars are not in excessively rapid rotation. It would seem then that the Stark effect or some cause not yet considered is responsible for the shallow absorption lines in early type stars.

If we may make the assumption that the total energy absorbed by a given number of atoms is the same in the presence, or absence, of an electric field or of other causes of broadening of the lines, then we may compare the observed areas of the contours with the calculated ones from Unsöld's equation,

$$(\lambda-\lambda_0)^2 = NHf\,\frac{2\pi e^4\lambda_0^2}{3m^2c^4}\left(\frac{1-I}{I}\right),$$

to obtain the effective number of atoms (NHf) producing the line. This has been done for all of the lines, and the results appear in Table I. Columns 1 and 2 contain the names and spectral types of

TABLE I *A*—*Continued*

No.	9	10	11	12	13	14	15	16	17	18
			Hβ					*Hγ*		
	$(\lambda-\lambda_0)$ Calc.	$(\lambda-\lambda_0)$ (Obs.)	(O−C)	I_0	$\frac{I_0}{(\lambda-\lambda_0)}$	$(\lambda-\lambda_0)$ Calc.	$(\lambda-\lambda_0)$ (Obs.)	(O−C)	I_0	$\frac{I_0}{(\lambda-\lambda_0)}$
51	7.5	12.2	4.7	55	4.5	13.1	19.8	6.7	65	3.3
52	13.6	20.1	6.5	67	3.3	13.1	19.8	6.7	66	3.3
53	13.0	19.6	6.6	69	3.5	8.0	13.0	5.0	67	5.2
54	4.1	8.8	4.7	52	5.9	7.0	10.0	3.0	>50	.....
55	7.5	12.2	4.7	>49	> 4.0	3.7	8.0	4.3	32	4.0
56	8.6	14.2	5.6	>52	> 3.7	7.0	13.0	6.0	52	4.0
57	9.6	16.2	6.6	70	4.3	8.4	13.3	4.9	57	4.3
58	6.4	9.8	3.4	>56	> 5.7	9.2	14.8	5.6	>55	.....
59	4.8	8.3	3.5	79	9.5	5.0	9.2	4.2	>61	.....
60	3.0	3.0	0.0	>90	>30.0	.....	.....	.....	.....	.....
61	2.5	3.5	1.0	80	22.8	2.2	2.5	0.3	84	33.5
62	1.8	2.9	1.1	60	20.7	.....	.....	.....	.....	.....
63	1.0	2.3	1.3	60	26.1	1.2	2.0	0.8	65	32.5
64	1.0	1.5	0.5	66	44.0	.....	.....	.....	.....	.....

the stars, and column 3 gives the adopted absolute magnitude; whenever possible, the trigonometric parallax has been used. Columns 4 and 5 contain the areas or total absorption of the line, respectively, for $H\beta$ and $H\gamma$. The unit of area is complete absorption over the width of 1 A. The respective values of log NHf are found in the next two columns. An inspection of columns 6 and 7 will show that the log NHf for $H\gamma$ is larger than for $H\beta$. These differences are given in column 8, the sign of the quantity being that for $H\gamma - H\beta$. The mean value of Δ log NHf is +0.23, which corresponds to a ratio of $H\gamma/H\beta$ equal to 1.7. The value of Δ log NHf varies over a considerable range, but there seems to be no correlation with the known physical properties of the stars, except for a slight tendency to decrease for the later spectral types.

The fact that log NHf is larger for $H\gamma$ than for $H\beta$ shows that the value of f given by theory[1] must be modified in its application to stellar problems. This difficulty may be partially eliminated when one considers a recent paper by Unsöld[2] where he shows that the value of f must be modified by a factor. However, it still seems that the modified value of f will be a function of the physical prop-

[1] Sugiura, *Journal de Physique*, **8**, 113, 1927.

[2] *Zeitschrift für Physik*, **59**, 353, 1930.

erties of the atmosphere since the variation of $\Delta \log NHf$ among the stars is larger than can be attributed to errors.

In order to obtain an idea of the broadening of the hydrogen lines due to causes other than the abundance of atoms, the half-widths of the lines at 10 per cent absorption have been calculated from the determined value of NHf. These half-widths are given in column 9 and 14 of Table I, respectively, for $H\beta$ and $H\gamma$. In columns 10 and 15 are the observed half-widths, and columns 11 and 16 contain the excess of the observed over the calculated widths.

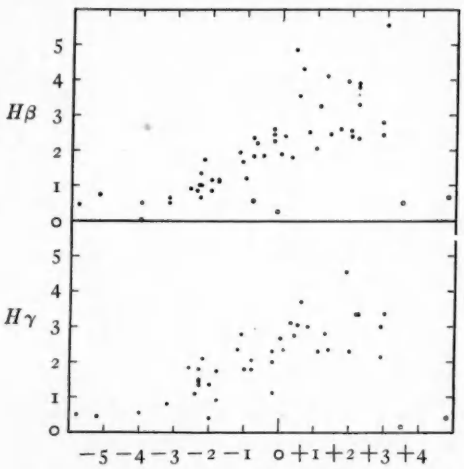

FIG. 3.—Correlation of the residual (observed minus computed) half-widths of the hydrogen lines with absolute magnitudes of the stars. The ordinates are angstrom units and the abscissae are absolute magnitudes.

If this excess is due to the Stark broadening, there should be a correlation with the absolute magnitude of the star. However, one should not expect this correlation to be a simple one, for the effects of a mol-electric field are proportional to the density of the charged particles, which in turn is proportional to the temperature and the pressure. The differences of absolute magnitude are mainly differences of pressures in the atmospheres of the stars. Hence, a correlation with absolute magnitude leaves out the temperature as an independent variable. If sufficient data were available the desirable correlation would be three-dimensional with the co-ordinates: amount of broadening, temperature, and pressure.

In the absence of sufficient data the correlation of absolute magnitude with the excess widths of the lines is shown in Figure 3 for $H\beta$ and $H\gamma$. There is a definite tendency for the observations to fall on a smooth curve, but in the later spectral types the dispersion is quite large, the deviations being marked for some of the stars of lowest

temperatures. These stars, which are plotted as circles, fail in the next correlation discussed.

Miss Emma Williams (*loc. cit.*), in a discussion of the contours of the hydrogen lines in class A stars, finds a correlation between the width and depth of a line with the luminosity of the star, the lines being progressively wider for their depth in passing toward the less luminous stars. The widening is interpreted as a manifestation of the Stark effect since the less luminous stars have larger pressures and hence a larger number of charged particles per unit volume.

For the stars under discussion the ratio of the half-width $(\lambda - \lambda_0)$ of the line at a depth of 10 per cent absorption, to its central intensity, I_0, has been determined and listed in Table I, columns 13 and 18, respectively, for $H\beta$ and $H\gamma$. Since the values of the ratio for $H\beta$ and $H\gamma$ are roughly the same, the means for the two lines have been taken, and in Figure 4 they are plotted against the absolute magnitude. An inspection of the plot shows that most of the observations group along a smooth curve, which is nearly flat between absolute magnitudes $+3$ and 0, and increases for the stars brighter than zero. The more luminous stars of class A and the stars of classes B and O fall on the part of the curve of greater slope.

In view of the uncertainties of the adopted absolute magnitudes of the early type stars the correlation seems to be quite satisfactory. In making the diagram no account was taken of the effect of rotation upon the line. This would cause the observation to fall below the curve, as is the case of 39 λ Orionis at $M = -2.1$. Another star for which rapid rotation is indicated is 19 q Tauri, which falls on the curve when the adopted absolute magnitude is used. However, if one takes the parallax as 0″.005, which was determined by Hertzsprung[1] for the Pleiades by a comparison of the color-luminosity-curve with stars of known parallax, we obtain the absolute magnitude of -2.1 for 19 q Tauri. This would put the observed point considerably below the curve in Figure 4.

The correlation is used to determine the absolute magnitudes of some of the early stars for which there are no parallaxes. These are as follows: 30 τ Canis Majoris, -2.0; 10 Lacertae, -3.0; 22 τ Herculis, -0.3; 4 γ Corvi, -0.9; and 30 η Leonis, -2.8. The following

[1] *Monthly Notices*, **89**, 660, 1929.

absolute magnitudes, instead of those adopted, are indicated for 47 ρ Leonis, -3.5; 22 λ Cephei, -2.5; 44 ζ Persei, -4.2; and 21 ϵ Canis Majoris, -3.9.

There are several stars indicated in the figure whose ratios, $I_0/(\lambda-\lambda_0)$, depart markedly from the smooth curve. They are stars of class F5 or later. Two of the class F5 stars, 41 Cygni and 33 α Persei, conform to the curve. The deviations are such as to indicate a small Stark effect, and perhaps this is what one should expect in the low-temperature stars even when the density is quite high, as in the

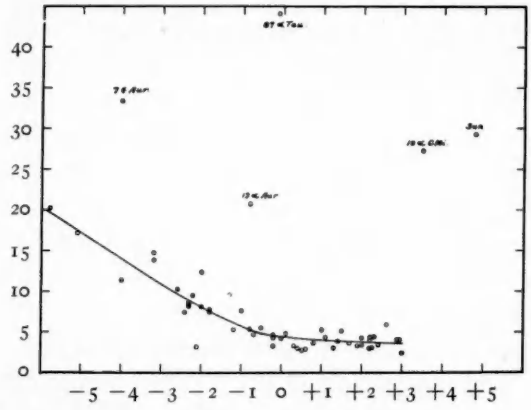

FIG. 4.—Correlation of the ratio, $I_0/(\lambda-\lambda_0)$, with absolute magnitude. The abscissae are absolute magnitudes.

dwarfs. The breakdown of the correlation occurs in the spectral types where the number of ions are decreasing rapidly due to the temperature and pressure becoming unfavorable for the dissociation of atoms of low ionization potentials.

In the papers cited above Milne has developed generalized formulae for the theory of thermal ionization as applied to the atmosphere of a star. For hydrogen we use his "*Problem III. The atoms in question are just becoming ionized in the presence of an excess of atoms already once-ionized.*" The resulting formula for the number of normal atoms in a given excited state is

$$N_0^{(s)}=\frac{2\epsilon}{mg}A_0^{(s)}\left[P_0-K_1 ln\left(1+\frac{P_0}{K_1}\right)\right], \qquad (1)$$

where ϵ is the abundance by weight; m, the mass of the atom; g, the value of gravity; $A_o^{(s)}$, the ratio of the number of neutral excited atoms in the given state to the number of neutral atoms; P_o, the pressure at the level considered; and K_1, the ionization constant. In the case where κ, the absorption coefficient, varies with the pressure, the value of the pressure at the level considered is calculated by the equation

$$P_o = \sqrt{\frac{\tau_0 g \left(\frac{T}{10^4}\right)^{9/2}}{a}}, \qquad (2)$$

where τ_0 is the optical depth of the level; a is a constant; and the other symbols have their usual meaning. For $\tau_0 = \frac{1}{3}$ and $a = 0.85$

TABLE II

(P_o in dynes, for $a = 10^{-3}$ and $\tau_0 = \frac{2}{3}$)

T	$g=10^{-1}$	$g=10$	$g=10^3$	T	$g=10^{-1}$	$g=10$	$g=10^3$
5000°.....	1.714	17.14	171.4	14,000.....	17.40	174.0	1740
6000.....	2.587	25.87	258.7	15,000.....	20.32	203.2	2032
7000.....	3.656	36.56	365.6	16,000.....	23.49	234.9	2349
8000.....	4.937	49.37	493.7	17,000.....	26.93	269.3	2693
9000.....	6.438	64.38	643.8	18,000.....	30.62	306.2	3062
10,000.....	8.160	81.60	816.0	19,000.....	34.58	345.8	3458
11,000.....	10.11	101.10	1011.0	20,000.....	38.82	388.2	3882
12,000.....	12.30	123.0	1230	25,000.....	64.13	641.3	6413
13,000.....	14.71	147.1	1471	30,000.....	96.66	966.6	9666

Milne obtained a series of curves representing the variation of $N_o^{(s)}$ with the temperature. These are shown by the dotted lines in Figure 5. Milne's computations give the maximum number of atoms as about 10^{15}. This is about one-ten-thousandth the value of NHf obtained for the hydrogen lines of Sirius, and is even smaller than the value of NHf for the stars of this list having the faintest lines. If we take the value of the constant, a, in equation (2) as 10^{-3} instead of 0.85 we obtain better agreement. In the expression for the coefficient of opacity, a is the constant, and the value of 10^{-3} is in fair agreement with the constant used in Kramers' formula.

Using this value of a and the optical depth τ_0, as $\frac{2}{3}$ (corresponding to the extreme wing of the absorption line), the pressures obtained from equation (2) are those given in Table II for g equaling 10^{-1},

10, and 10^3. From equation (1) the number of atoms have been computed, assuming an abundance of 10 per cent by weight. This is perhaps a little small in view of the determination of the abundance of hydrogen in the sun by Russell. The resulting curves are shown by the continuous lines in Figure 5.

To compare the observations with these curves, separate plots are made for each $H\beta$ and $H\gamma$. The values of log NHf are used instead of log NH, owing to the uncertainty of the value of the quantity f. The adopted temperature scale is that given by Russell, Dugan, and Stewart.[1]

The observations plotted in Figure 5 are in disagreement with the theoretical curves. On the low-temperature side of the maximum the slope of the observed curve is much smaller than the calculated one, the distribution of the giants and dwarfs having been taken into consideration. The observations indicate a tenfold increase in the number of atoms from 5000° to 7500° while the theory requires a thousand-fold or greater increase.

On the high-temperature side of the maximum the observations do not show as rapid a change with temperature, there being a tenfold to a hundred-fold decrease in the number of atoms between the temperatures of 10,000° and 30,000°. The theoretical curves show a small decrease between the maximum and 15,000°, but toward higher temperatures the curve again increases. This[2] is best shown by a consideration of equation (1), in which, for high temperatures, we may expand $ln\ (1+[P_0/K_1])$ into a series and use only the first two terms. The result is a simplified formula for the number of atoms:

$$N_0^{(s)} = \frac{2\epsilon}{mg} A_0^{(s)} \frac{P_0^2}{2K_1} . \tag{3}$$

Substituting the values of $A_0^{(s)}$ and of P_0 in terms of T, and designating all of the constants by C, we obtain

$$N_0^{(s)} = C \frac{e^{\frac{-(\nu_1 - x_1^s)}{kT}} T^2}{e^{\frac{-\nu_1}{kT}}} , \tag{4}$$

[1] *Astronomy*, **2**, p. 753.

[2] These remarks I owe to discussions with Professor A. Pannekoek and Mr. W. W Markowitz.

where

$$C=\frac{2\epsilon}{mg}\cdot\frac{q_o^{(s)}}{q_o}\,\frac{\tau_0 g}{a}\,\frac{1}{10^{18}}\,\frac{1}{2\,\dfrac{q_1}{q_o}\,\dfrac{(2\pi m_e)^{3/2}}{h}\,k^{5/2}}\,. \tag{5}$$

From equation (4) we see that $N_o^{(s)}$ increases with the temperature for high temperatures. For hydrogen the value of C is 1.916×10^{8}. A check computation of $N_o^{(s)}$ using equation (4) gives 5.44×10^{17}, which is in exact agreement with that obtained from equation (1).

Milne[1] has recently shown that on the high-temperature side of the maximum of a given line there is no effect of absolute magnitude which is attributable to the abundance of atoms although there may be one due to the electric fields. He has termed this the "null effect." It may be readily seen by referring to equations (4) and (5) that the quantity g cancels at high temperatures. Milne attributes any change in line width between two stars of the same temperature (higher than the maximum) to causes other than atmospheric thickness or ionization, e.g., Stark effect or rotation. The "null effect," however, neglects the second stage of ionization, and if the ionization potential is not too high above the previous ionization potential the correction to be applied for the second stage of ionization will have an effect which is dependent upon g. This would of course apply to atoms other than hydrogen. The effect is no doubt small for there are few examples where in a given spectrum the lines are present from three atoms of the same species but in different states of ionization.

If it is legitimate to determine the number of the atoms forming an absorption line from the area of the contour when it is distorted by the Stark effect and rotation of the star, we then have a test of the "null effect." In Table I there are three supergiant stars, at or past the maximum, each of which gives values of log NHf smaller than the other stars of their classes. These stars are: 50 α Cygni, A2p; 19 β Orionis, B8p; and 67 Ophiuchi, B5. The star 30 η Leonis, A0p, which from a preceding correlation is the brightest A0 star, has the smallest log NHf of that class. Taking the mean contours of $H\gamma$ given by Misses Payne and Williams for several B5 and cB5

[1] *Harvard College Observatory Bulletin*, No. 870, 1, 1929.

stars, the respective logarithms of *NHf* are 18.37 and 17.69. The stars with the *c*-characteristics are considered to be supergiants.

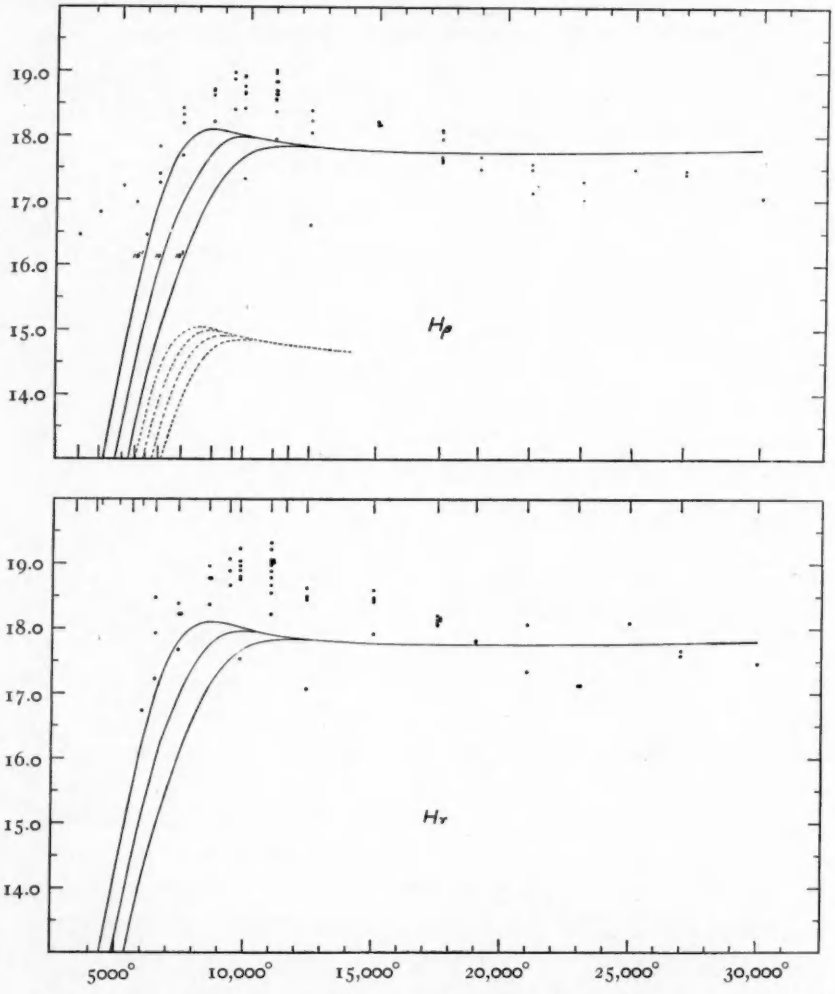

FIG. 5.—Variation of log *NH* with temperature. The dots are the observed values of log *NHf*. The dotted curves are those computed by Milne with $a=0.85$, $\epsilon=0.05$, and $\tau_0=1/3$ for the surface gravities 10, 10^2, 10^3, and 10^4. The continuous curves are computed for $a=10^{-3}$, $\epsilon=0.10$, and $\tau_0=2/3$ for surface gravities 10^{-1}, 10, and 10^2. The absorption coefficient varies as $P/T^{9/2}$.

A possible reason for this discrepancy with the theory is that the gas pressure used to calculate the number of atoms is not the effec-

tive pressure. If a correction which is some quantity proportional to a higher power of the temperature, but not variable with g, can be subtracted from the pressure we then have a means of overcoming the difficulty. The correction would be much more effective for the supergiant stars, for in them the gas pressure is relatively smaller. This would cause the curves for various values of g shown in Figure 5 to cross one another; the ones for smaller values of g would be decreasing more rapidly. Such a correction to the pressure might be that due to general radiation pressure, which is proportional to the fourth power of the temperature. Such a correction would also eliminate the difficulties of the value of $N_o^{(s)}$ increasing toward infinity with the temperature as is shown by equation (4).

NOTE ADDED MARCH 28, 1930

Since the foregoing paper has been in the hands of the printer, an article on "Continuous Absorption" by J. A. Gaunt has appeared in the *Proceedings of the Royal Society*, **126,** 654, March 3, 1930, and contains the following note:

> It may be well to add a remark on the variation of the continuous absorption coefficient with temperature in the theory of absorption lines. On the basis of Kramers' formula, the mean absorption coefficient varies as $P/T^{9/2}$, where P is the electron-pressure. But here a T^3 arises in the denominator through averaging ν^3 in (4.0). When we consider, as we should, the absorption at a definite frequency, a more likely variation is as $P/T^{3/2}$.

Since the variation with the temperature of log $N_o^{(s)}$ for hydrogen was computed from Milne's formulae in which the coefficient of absorption varied as $P/T^{9/2}$, it has been recomputed, making use of the suggestion of Gaunt. With the use of the same values of ϵ, τ_0, and a, and of g equal to 10, the resulting pressures are

T.....	5000°	6000°	7000°	8000°	9000°	10,000°	
P.....	51.4	55.6	62.3	68.6	75.4	81.6	dynes
T.....	11,000°	12,000°	13,000°	14,000°	15,000°	16,000°	
P.....	87.5	93.5	99.3	105.0	110.8	116.1	dynes
T.....	17,000°	18,000°	19,000°	20,000°	25,000°	30,000°	
P.....	121.5	127.0	132.0	137.5	162.2	186.1	dynes

instead of those in Table II.

The curve of the variation of log $N_o^{(s)}$ with T for the new pressures

in shown in Figure 5*a*, which contains the curves and observations in the upper half of Figure 5 for comparison. The new curve fits the observations much more satisfactorily, especially on the side of the maximum toward higher temperatures. The supergiant stars, however, disagree, as was noted. On the side toward low temperatures there is still considerable deviation of the observations from theory, and it will be desirable to make a comparison with a greater number of observations for the stars of late types. The curve and the obser-

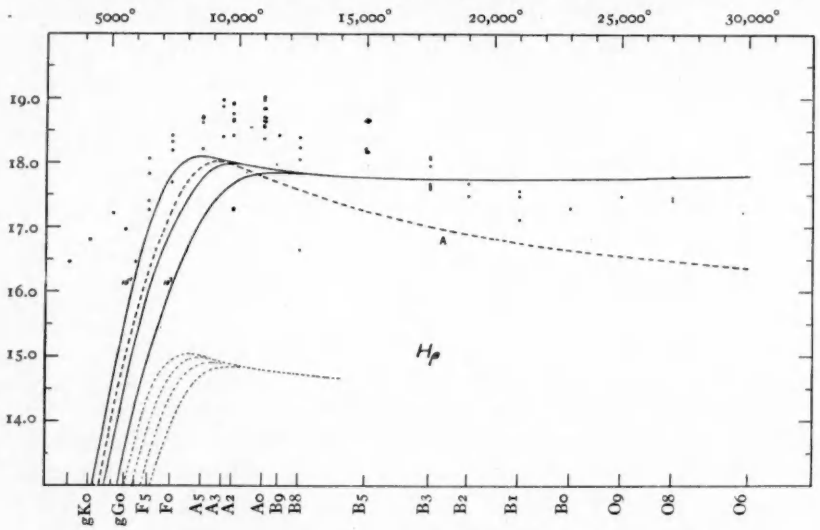

FIG. 5*a*.—Curve *A* has been computed assuming that the variation of the absorption coefficient is as $P/T^{3/2}$ instead of as $P/T^{9/2}$. The constants are the same as the former curve for $g=10$.

vations are not in coincidence, owing to the uncertainty of the constant factors; ϵ, the abundance; a, the constant in the coefficient of opacity; and f, the oscillatory strength used in the formula of Unsöld. The variations in these factors shift the curves, or observations, as a whole along the vertical axis of the diagram.

The objection raised to Milne's formula for log $N_0^{(s)}$, i.e., increasing indefinitely with the temperature, does not exist when Gaunt's suggestion of the temperature variation of the coefficient of absorption is used.

YERKES OBSERVATORY
February 28, 1930

ON THE RADIAL VELOCITY OF R LYRAE[1]

By ROSCOE F. SANFORD

ABSTRACT

Observations.—Table I gives thirty-nine radial velocities from one-prism spectrograms of R Lyrae obtained at Mount Wilson in 1917, 1923, 1924, 1925, and 1929.

Radial-velocity variation.—Consistent seasonal values of the phases of maximum and minimum velocity (Table II) are given by the formula

Phase = Julian Day of observation
$-[\text{J.D. } 2423600.000 + 43^{d}.48E + 6^{d}.5 \sin (2°.7E - 47°.2)]$ G.M.T.

The nature of the mean variation of the radial velocity is shown by the plots in Fig. 1, for Lick, Allegheny, and Mount Wilson observations. Mrazek's period does not fit the complete data.

The scattering of the observations, the difference in amplitude apparent in different series, and some difference in systemic velocity may indicate a velocity variation other than that corresponding to a period of 43.48 days. This, however, can hardly be the five-year variation in the center-of-mass velocity found by Mrazek. The most serious objections to this are (*a*) the presence of a systematic difference between the Allegheny and Mount Wilson wave-lengths that would reduce the amplitude of Mrazek's long-period variation to 5 km/sec. (one-half his value) and (*b*) the failure of the Mount Wilson observations of 1925 to verify a predicted minimum velocity.

Spectrum.—There are at least no outstanding spectral changes which follow the period of 43.48 days.

The variability in light of the star now designated as R Lyrae[2] was discovered by Baxendell in 1855.[3] The very numerous photometric observations which have been accumulated since then indicate that, if periodic, the variation in light takes place in a cycle of 45± days, with an amplitude that does not exceed 0.7 of a magnitude and is often not more than 0.3 or 0.4. No one, however, has been able to find a law of variation which is competent to represent at all satisfactorily the complete photometric data. The star is interesting because it is one of those variables whose periods lie between those of the Cepheids and the long-period variables.

The variation in the radial velocity of R Lyrae was announced by Adams, Joy, and Sanford[4] from measures of spectrograms made at Mount Wilson. Daniels[5] had previously concluded that eighteen ob-

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 394.

[2] B.D.+43°3117, H.D. 175865, Boss 4814, α $18^{h}52^{m}3$, δ +43°48′8 (1900); sp. Mb; mean mag. 4.3.

[3] *Monthly Notices, Royal Astronomical Society*, **16**, 201, 1856.

[4] *Publications of the Astronomical Society of the Pacific*, **36**, 137, 1924.

[5] *Publications of the Allegheny Observatory*, **2**, 125, 1910.

servations, which he made at Allegheny over an interval of 39 days in 1910, gave no evidence of a variation in radial velocity because the probable error of their mean was quite as small as could be expected in view of the type of photographic plate and the dispersion employed. The only additional observations thus far published are those from the Lick Observatory.[1] These comprise three groups, 1899–1901, 1907, and 1912, and are all from measures of three-prism spectrograms.

Mrazek[2] has discussed the Lick radial velocities of 1899–1901 and 1907 and those obtained at the Allegheny Observatory and certain contemporary observations of light variation. He concludes that both velocity variation and light variation are satisfied by a period of 41.3 days, and that light maximum precedes velocity minimum in the three cases by 4.6, 6, and 4.6 days, respectively. He also finds a difference of nearly 10 km/sec. in the systemic velocity, which suggests a periodic change. Meridian-circle observations of the right ascension of R Lyrae made at Munich[3] over an interval of years furnish values of $\Delta\alpha$ which not only give the proper motion in that coordinate but also indicate a periodicity of five years in the motion. He draws attention to this as possible evidence of orbital motion about an invisible companion and then derives a curve for the variation in the velocity of the system on the assumption that the Allegheny observations are at the minimum and the Lick observations of 1901 at the maximum of this velocity variation and correspond to the points of maximum and minimum, respectively, of the $\Delta\alpha$-curve. Intermediate values of the radial velocity are inferred by interpolating values on the supposition that the form of the velocity-curve is the same as that of the $\Delta\alpha$-curve. The result is a variation in the velocity of the system with a period of five years and a semi-amplitude of 4.55 km/sec. The form of the velocity-curve corresponds to motion in an orbit with an eccentricity of 0.52.

It is now possible to add thirty-nine more radial velocities, obtained at the Mount Wilson Observatory, mostly with the 60-inch reflector, in 1917, 1923, 1924, 1925, and 1929. Practically all are from spectrograms having a dispersion of 37 A per millimeter at $H\gamma$,

[1] *Publications of the Lick Observatory*, **16**, 278, 1928.

[2] *Astronomische Nachrichten*, **236**, 281, 1929.

[3] *Ibid.*, **209**, 55, 1919.

which were reduced with the aid of a table of stellar wave-lengths derived by Merrill[1] from measures of absorption lines in the spectra

TABLE I

MOUNT WILSON OBSERVATIONS OF R LYRAE

No.	Plate	Date	G.M.T.	Vel.	Wt.
				km/sec.	
1	γ 5804	1917 May 11	23^h40^m	−26.7	.....
2	5957	July 7	20 20	27.6	.....
3	6011	July 30	20 59	29.0	.....
4	11752	1923 May 2	22 4	30.4	.....
5	11764	May 5	21 43	30.4	.....
6	11769	May 6	20 5	34.8	.....
7	11813	May 28	22 26	23.8	.....
8	11819	May 29	21 12	21.8	.....
9	11901	June 28	19 58	20.8	0.5
10	11906	June 29	19 30	26.9	.....
11	11905	Aug. 3	19 15	29.0	.5
12	12088	Sept. 1	15 15	27.6	.....
13	12172	Oct. 3	14 42	28.4	.....
14	C 2497	Oct. 22	15 10	31.5	.....
15	γ 12242	Oct. 28	14 36	28.2	.....
16	12298	Nov. 25	14 58	30.0	.....
17	12642	1924 May 15	21 1	24.1	.....
18	12644	May 15	22 32	23.3	.....
19	12745	June 18	19 38	27.2	.....
20	12783	July 13	16 14	32.4	.....
21	12893	Aug. 20	15 28	31.7	.5
22	12938	Sept. 10	16 2	25.6	.....
23	13022	Oct. 15	14 53	29.0	.5
24	13094	Nov. 11	15 01	27.7	.....
25	13264	1925 Feb. 10	1 30	26.5	.5
26	13305	Mar. 11	1 10	22.7	.....
27	13347	Apr. 3	0 57	22.6	.5
28	13441	June 7	21 4	26.4	.....
29	13516	July 5	16 57	34.2	.....
30	13621	Aug. 6	16 27	25.0	.....
31	13710	Sept. 8	14 57	20.9	.....
32	13729	Sept. 23	15 10	33.9	.5
33	16600	1929 May 22	23 43	25.0	.5
34	16603	May 23	19 45	28.2	.5
35	16606	May 23	23 52	28.8	0.5
36	16635	June 16	22 10	31.6	.....
37	16723	July 18	23 52	30.4	.....
38	16832	Aug. 20	19 27	26.1	.....
39	16900	Sept. 12	14 47	−32.1	.....

of Me-type stars. All the results for lines on a single plate are very consistent. About half of the velocities are the means of two or more measures. The agreement for repeated measures was found to be

[1] *Mt. Wilson Contr.*, No. 265; *Astrophysical Journal*, **58**, 195, 1923.

such that little would have been gained by remeasuring spectrograms for which the velocity already derived depended on a large number of lines. Table I gives the data of observation and measurement for this series of velocities. The only comment required is that plates of inferior quality which still seemed useful were assigned weight one-half; the remainder have unit weight.

It is evident from this table and from the Lick and Allegheny observations that the variation in radial velocity is small and that an adequate discussion therefore calls for spectrograms of at least as high an order of accuracy as those made at the Lick Observatory. Nevertheless it seems best to present the additional Mount Wilson data and consider their consistency with Mrazek's conclusions.

TABLE II

PHASES OF MAXIMUM AND MINIMUM VELOCITY

Series	Observatory	Phase of Max.	Phase of Min.
1901	Lick	17^d	38^d
1907	Lick	19	38
1910	Allegheny	18	37
1912	Lick	16	35
1923	Mt. Wilson	19	35
1924	Mt. Wilson	16	33
1925	Mt. Wilson	15	35
1929	Mt. Wilson	20	40

Julian Day 2423600.000, which is approximately in the middle of the interval covered by the Mount Wilson observations, is the epoch to which they have been reduced by means of Mrazek's period. When thus reduced to a single cycle and grouped according to seasons (1923, 1924, 1925, and 1929), some evidence of a periodicity appears, but the phases of maximum and minimum velocity for the different seasons do not agree. Since this disagreement is very marked between 1923 and 1924, the period evidently needs considerable change. Trials with periods smaller than 41.3 days proved futile. The best agreement was found with 43.48 days, which also gives fairly consistent results for the phases of maximum and minimum velocity for the seasons involved in the earlier observations (1901, 1907, 1910, and 1912). There remains, however, some spread in these phases, and, when plotted, they show a periodic variation which can

be represented by a sine term. If this variation be taken into account by computing anew all phases by means of the formula

$$\text{Phase} = \text{Julian Day of observation} - [\text{J.D. } 2423600.000 + 43^d48E + 6^d5 \sin (2°7E - 47°2)] \text{ G.M.T.}, \quad (1)$$

the results for maximum and minimum velocity for the different seasons are approximately as given in Table II.

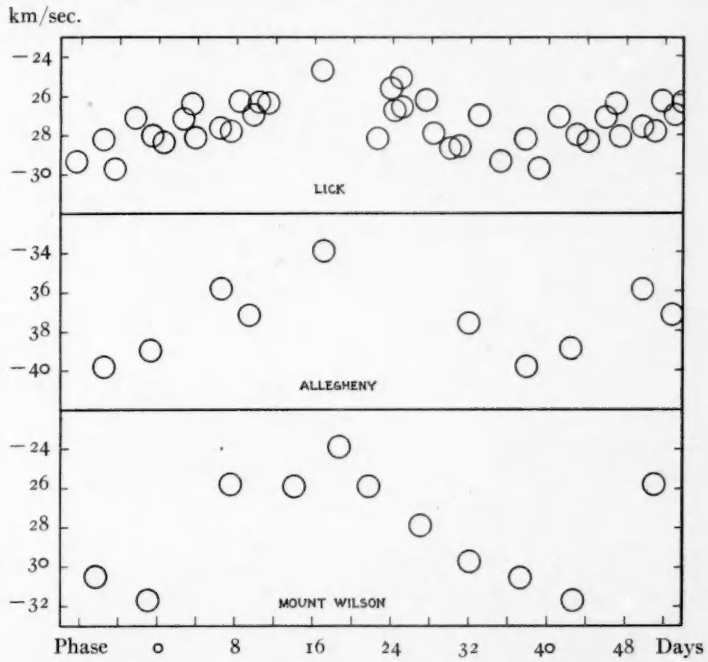

FIG. 1.—Radial-velocity diagrams derived by means of formula (1) from the Lick observations of 1901, 1907, and 1912, the Allegheny observations of 1910, and the Mount Wilson observations of 1923, 1924, 1925, and 1929. Normal places are plotted for Allegheny and Mount Wilson, but individual velocities for the Lick Observatory measures. The radius of the circles corresponds to 0.5 km/sec., which is of the order of the probable error of a plotted velocity.

No further attempt at refinement with this mode of computing the phases seems warranted, since, for the low-dispersion observations at least, the accidental errors in the radial velocities are probably a large percentage of the total amplitude involved. Figure 1 shows the variation in radial velocity derived from the Lick, Al-

legheny, and Mount Wilson observations, respectively. On account of their relatively higher accuracy, individual radial velocities are plotted on the Lick Observatory diagram. The Allegheny observations cover less than a single period, and each plotted point is the mean of three successive values. The points on the Mount Wilson diagram are means of velocities falling within intervals of five days in phase, the grouping being that shown in Table III.

It is evident from the Lick Observatory diagram, where the determinations are of relatively high weight, that the scatter is more than would be anticipated if simple periodic velocity variation alone is involved, and the individual observations made at both Allegheny

TABLE III

MOUNT WILSON NORMAL PLACES

No.	Phase	Radial Vel.	Observation No. from Table I, Col. 1
		km/sec.	
I	$7^d.449$	−25.8	9, 10, 19, 22
II	14.033	25.8	13, 26, 28
III	18.576	23.9	7, 8, 31, 33, 34, 35
IV	21.602	25.9	16, 17, 18, 38
V	26.896	27.9	12, 21, 24, 25
VI	32.000	29.7	14, 30, 32, 37
VII	37.236	30.5	4, 5, 6, 15, 20, 27
VIII	42.588	−31.7	11, 23, 29, 36, 39

and Mount Wilson bear this out. It is hard, therefore, to escape the conclusion that other variations, either erratic or periodic, may also be involved.

As already stated, Mrazek has concluded that a variation in the velocity of the center of mass with a period of five years is also present. Although a long-period variation may be involved, several points may be urged against Mrazek's value. In the first place, his long-period velocity-curve requires a minimum in 1925. The mean velocity for the eight observations made at Mount Wilson in 1925 is −26.6 km/sec., which is nearly 10 km/sec. higher than the predicted minimum. Second, the seasonal means are in substantial agreement except for the Allegheny observations. This suggests the possibility of a systematic difference between Allegheny determinations and those made at the Lick and Mount Wilson observatories. Such a

difference really exists, and explains at least a part of the discrepancy. Daniels has published the wave-lengths which he used; and a sufficient number of his lines appear in Merrill's list to show that, if he had used Merrill's wave-lengths instead of his own, his radial velocities would have been larger algebraically by approximately 5 km/sec. Proper attention was given in the comparison to the fact that Daniels' wave-lengths are on the Rowland system and Merrill's on the international system. It is safe to say, therefore, that, if the total amplitude of long-period variation in the γ-velocity is involved in the difference between Allegheny, on the one hand, and the Lick and Mount Wilson observations, on the other, it can only be about one-half what Mrazek inferred.[1] It seems equally evident that the period is not five years.

The amplitudes of the velocity variation of the Lick, Allegheny, and Mount Wilson observations differ somewhat, as may be seen from Figure 1, although in kilometers per second the differences are not large. Nevertheless, they may be taken as further evidence of a variation additional to that in the period of 43.48 days.

The small amplitude of variation found with this period, together with the scattering still remaining in the individual observations, may suggest to some that the real period has not yet been found. Such a possibility is recognized, but perhaps is offset by the following facts: (1) the period is of the order of magnitude found many times by photometric observers; (2) the three series are harmoniously periodic; and (3) the probable errors derived with the residuals from simple means of the respective series are larger than would be expected from spectrograms as easily measurable as these.

The Mount Wilson spectrograms have been examined for any outstanding changes in the spectrum synchronous with the period of velocity variation, but none was found.

While most photometric observers assign a period in the neighborhood of 45 days, no one has yet had complete success in devising a formula which represents the great mass of photometric data that

[1] The possibility that the difference of 5 km/sec. which still remains is a systematic difference of instrumental origin has not been overlooked, but other evidence is lacking. It hardly seems reasonable. It is equally unlikely that Campbell's K-term for stars of spectral class M (+4 km/sec.) affects the Lick and Mount Wilson values and not those of Allegheny.

has been accumulated. It has seemed preferable, therefore, to discuss the radial velocities independently of the photometric observations. When the photometric data are later brought into harmony some definite relation may be forthcoming.

Briefly, the radial velocities of R Lyrae show that in the mean, at least, they are consistently represented in phase by formula (1). The mean amplitude is small. The scattering of the observations, the difference in amplitude for the curves of different series, and the difference in systemic velocity given by the Allegheny observations and by the Lick and Mount Wilson measures suggest a superposed velocity variation which may be either erratic or periodic, but which cannot be established from the present observations. Spectral changes synchronous with the period, if present, are not at all outstanding.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
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REVIEWS

Matter, Electricity, Energy. By WALTER GERLACH. Translated from the second German edition by FRANCIS J. FUCHS. New York: D. Van Nostrand Co., 1928. Pp. xii+427. $6.00.

English-speaking students certainly will be much obliged to Dr. F. Fuchs for translating this excellent book. Professor W. Gerlach, of Tübingen, is one of the foremost German physicists, who has enriched modern experimental physics by several important discoveries and seems particularly qualified to write a general book on atomic physics. It is not a textbook, nor is it one of those numerous popular works which have appeared in the windows of the bookstores during recent years. It is addressed to those who, being acquainted with classical physics, wish to get an insight into the problems of modern atomistics, without being embarrassed by high mathematical and technical details. *Matter, Electricity, Energy* has been already adopted by German university teachers for the seminar work of graduate students. As such it will be of great use in American colleges since it gives a good starting-point for more advanced study.

Except for relativity Professor Gerlach's book covers practically all the more important problems of modern physics. In thirty chapters the author deals with atomic, electronic, and magnetonic theories and experimental data including X-rays, infra-red and electromagnetic spectra, photochemistry, crystal structures, etc. With especial care and ability the author treats the difficult and disputable problem of nuclear disintegration. The variant hypotheses of Sir Ernest Rutherford (the satellite view) and of Kirsch-Pettersson (the explosion theory) are presented in a very illuminating and comprehensive manner. The controversy between Millikan and Ehrenhaft on the fundamental unit of electricity and the existence of subelectrons is completely described with a full and fascinating account of the experimental side of the problem, including an analysis of the Stokes-Cunningham law and the intervening effect of Brownian motion. It is amazing what an enormous amount of physical data and theories are presented by the author in the pages of this volume. In only a few places does the reader feel a lack of proportion. The chapters on Bohr's theory and optical spectra of elements are rather short; perhaps the author considered these problems as more familiar to a general reader.

The concluding chapter on "Atomism and Macrocosmos" can hardly satisfy any curious reader; it is too short to be useful and does not give an account of the remarkable progress of modern astrophysics. Another defect of the present edition is that the English translation (1928) was made from the second German edition (1926), which was not revised by the author; thus the recent progress of physics (including quantum mechanics, data on penetrating radiation, etc.) has not been included in the book.

These defects, of course, cannot diminish the value of Professor Gerlach's work, for it will be easy to correct them in a new edition. The excellent book under review undoubtedly gives one of the best representations of the principles of modern atomistics and experimental results of atomic investigations. As such it is to be recommended to all college students in physics, as well as to the general reader.

B. P. GERASIMOVIČ

KHARKOV, U.S.S.R.

Atoms, Molecules and Quanta. By ARTHUR EDWARD RUARK and HAROLD CLAYTON UREY. New York: McGraw-Hill Book Co., 1930. Pp. xvii+790. Figs. 232. $7.00 net.

This volume constitutes a very considerable addition to modern physical literature. As a comprehensive and thoroughly up-to-date English text on quantum theory, primarily from the experimental point of view, it fills a very real need, as will be realized by anyone who has taught the subject. At the same time it should be a valuable reference-book for all workers in the field, containing, as it does, a large amount of accurate information, clearly displayed.

The first two-thirds of the book, which covers most of the detailed discussion of experimental material, is written from the standpoint of the older quantum theory of electron orbits, with, of course, the proper corrections added from the quantum mechanics which is later developed. No previous knowledge is assumed beyond that possessed by the average graduate student; the requisite advanced dynamics is developed as needed, though much of the book may be read without Hamiltonian mechanics. Atomic and molecular spectra of all types are thoroughly discussed in relation to theory, and there is an adequate treatment of critical potentials, the periodic system, and collisions of the second kind. Throughout, the emphasis is very properly placed on the results of calculations and their comparison with experiment rather than on methods of computation, though these are usually given. The remainder of the book (about two hundred pages) is devoted to the new quantum theory, commencing with the wave mechanics and proceeding through matrix mechanics to the

general-transformation theory. Without much detail of special calculations the theory is quite completely presented and most results of importance are included, although Dirac's theory of radiation and the spinning electron are omitted. Aperiodic phenomena are only briefly touched on, and the authors regret that lack of space made it impossible to include chapters on quantum phenomena within the nucleus and the new statistics.

The diligent student should certainly be able to acquire from this book a good working knowledge of quantum theory and an appreciation of what it can do. On the other hand, it may be more difficult for him to obtain a deeper insight into the physical meaning and logical consistency of the underlying principles; not only because critical and philosophical discussion has been cut to a minimum, but because the plan of the book makes the development of basic principles necessarily rather scattered. One might possibly suggest more discussion of fundamental experiments from the standpoint of wave mechanics, greater emphasis on the dual nature of matter and radiation, and a less abstruse introduction to the wave theory.

Printing and binding are excellent, except for a rather large number of typographical errors.

FRANK HOYT

RYERSON LABORATORY

An Outline of Stellar Astronomy. By PETER DOIG. London: Draughstman Publishing Co., 1927. Pp. vii+183. Figs. 24. $2.00.

An interesting and instructive compilation of material on the magnitudes, distribution, constitution, etc., of the stars, this book is written primarily for the layman, or for the student in other fields, who wishes a rapid survey of the recent work done on the subjects of which it treats. The treatment given is rather condensed, as is obviously necessary in a book of only 183 pages, yet the placing of as much of the information as possible in tabular form allows the book to contain a surprising amount. The text is, for the most part, entirely non-mathematical, but there are several appendices in which derivations are given for some of the simpler formulae.

One particularly commendable feature of the book is that it does not state as facts things that are only theories or hypotheses. The exposition is clear and concise, and of a nature easily understood by the layman. There is an excellent Bibliography at the end of each chapter, and an adequate Index to the book.

A. S. F.

PREPARATION OF ABSTRACTS

Every article in the *Astrophysical Journal*, however short, is to be preceded by an abstract which should be submitted by the author with the manuscript. In order that the abstract may aid the reader by furnishing an index and brief summary of the contents of the article, and may also be suitable for reprinting in an abstract journal, it should be of the type of those which have been appearing in the *Astrophysical Journal* since 1920. It is requested that the abstracts be prepared in accordance with the following:

DIRECTIONS AND RULES

1. *Notes.*—Read the article carefully, taking rough notes covering all the new information reported, keeping a specially sharp lookout for new incidental results and suggestions not directly related to the main subject.

Material not new need not be described; a valuable summary of previous work, however, should be noted.

2. *Subtitles.*—Write, first, a title describing the group of results forming the main contribution of the article, including all that belong together. If there are in addition results which do not come under that title, gather them into as few groups as possible and formulate a complete and precise title for each.

The subtitles should together form a complete index of the new information; that is, they should include every measurement, observation, method, suggestion, and theory which is presented as new and of value in itself. They should be complete in themselves and independent of the main title of the article.

3. *Text.*—Write a paragraph summarizing the main group of results and including the corresponding subtitle either all at the beginning or with parts scattered through the text; and then do likewise for the other groups.

A separate paragraph should be used for each distinct subject involved, but no more than necessary. All material which can easily be grouped together under a single title should be summarized in the same paragraph. Parts of subtitles may be scattered through the text, but the subject of each paragraph must be given at the beginning.

The text should summarize the author's conclusions and should transcribe all numerical results of general interest, including all that might be looked for in a table of astronomical and physical constants, with an indication of the accuracy of each. It should give all the information that anyone not a specialist in the particular field involved might care to have in his notebook.

Complete sentences should be used except in the case of subtitles. The abstract should be made as readable as the necessary brevity will permit. Italicize subtitles but no other words or phrases.

4. *Final checking.*—Re-read the article so as to check the abstract and correct any omissions and mistakes; read the subtitles by themselves to see that they properly index the information; and read the abstract to see whether it cannot be condensed and its English be improved.